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The Empirics of the Labour Theory of Value: Reply to Nitzan and Bichler

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Abstract

The labor theory of value, originated in the classics and reformulated by Marx, has found support in numerous empirical works during the last thirty years. In many economies, sectors in monetary terms are highly correlated with them in terms of labor values. In his book Capital as Power (2009), and in a subsequent discussion online, Jonathan Nitzan and Shimshon Bichler argue that such results are invalid because the calculations do not use labor value variables but two monetary variables are correlated. Nitzan and Bichler also argue that are spurious correlations by the presence of a third variable. This article refutes both critics and consequently reinforces the empirical support for the theory of labor value.

Key words: labor theory of value, empirical verification and Marxist theory, spurious correlation.

JEL **Classification**: B510, C18.

Introduction

Over the years since the pioneering work of Anwar Shaikh (1984), the present authors and many other researchers have presented findings of a close correlation between monetary value and labour content across the industrial sectors of capitalist economics, and have argued that these findings support the classical labour theory of value.[1] In their book *Capital as Power* (2009), and in

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1 See for example Petrovic (1987), Ochoa (1989), Valle Baeza (1994), Tsoulfidis and Maniatis (2002), Zachariah (2006), Tsoulfidis and Mariolis (2007), Fröhlich (2010b), Cockshott and Cottrell (1997a; 1998; 2003).

subsequent online discussion,[2] Jonathan Nitzan and Shimshon Bichler (2009) argue that such studies are fatally flawed, on two main grounds.

1. The empirical studies in question do not use actual labour-time data to estimate labour content but instead use monetary data from the input-output tables. Those of us who have claimed evidence for a close correlation between labour values and prices are guilty of circular reasoning since we presume what we must show: that it is possible to work backwards from money to labour time.
2. The correlations presented in the literature are spurious since they do not take into account industry scale. The observed price-value correlations are, they claim, an artifact of the differing scale of industries, with "large" industries naturally showing both a high aggregate price of output and high labour content.

On the first of these points Nitzan and Bichler (2009: 94) write that, in order to examine the relationship between prices and values, "two things must be known beforehand: prices and values. And yet it turns out that these seemingly trivial magnitudes are not so easy to 'know' and that, contrary to their explicit proclamation, the empirical studies do not appear to even try to correlate prices and values." They further state, "To our knowledge, all Marxist models that purport to correlate prices with values do no such thing. Instead of correlating prices with values, they in fact correlate prices with . . . prices!" (p. 96).

On the second point, they criticise the practice of correlating aggregate sectoral prices and values in these terms:

> Correlations measured in this way reflect the co-variations not only of unit prices and values, but also of their associated quantities. Now, note that the unit value and unit price of each sector are multiplied by the same output. This fact means that, all other things being equal, the greater the size variability of output across the different sectors, the tighter the correlation between their total price and total value. And since different sectors do vary in their output size, the common result is to make the overall correlation bigger than the underlying correlation between unit prices and values. The extent of this impact is revealed when sectors are controlled for their size: the value price correlations usually drop sharply, often to insignificant levels (Nitzan and Bichler, 2009: 95).

We respond to both of these arguments below. Section 2 addresses the charge that proponents of the labour theory of value have failed to measure labour values, and section 3 presents a rebuttal of the claim that sectoral price-value correlations are spurious. Section 4 gives a brief conclusion.

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[2] See Cockshott, Bichler and Nitzan (2010).

REAL QUANTITIES VERSUS MONETARY MAGNITUDES

We first probe Nitzan and Bichler's argument that researchers who have claimed to examine the empirical correlation of prices and Marxian values in fact do no such thing, their analysis being confined to the interrelations of prices at various levels. Three sorts of grounds may be given for such a critique.

1. Marx defined the value of a commodity as the quantity of socially necessary abstract labour time required (directly and indirectly) to produce that commodity. But socially necessary abstract labour time (SNLT) is not, even in principle, an observable magnitude.
2. Even if one fudges the issue by identifying SNLT with the clock hours of labour performed, such data are not available and those claiming to test the labour theory of value have actually used industry wage bills as a proxy for direct labour hours.
3. To calculate the labour time *indirectly* required for a commodity's production one needs data on the "technical coefficients" that link the sectors of the economy. In principle these coefficients should give the amount of industry *j*'s product, measured in natural units, required to produce one unit of industry *i*'s product (where the indices *i* and *j* range across the n industries that compose the economy). But in fact the available input-output tables give coefficients based on monetary magnitudes: the dollars-worth of input from industry *j* required to produce a dollar's worth of output in industry *i*.

The first of these points is explicitly flagged by Nitzan and Bichler; witness their assertion that "this quantum [socially necessary abstract labour time] is impossible to measure" (2009: 96). The other two points seem to be implicit in their claims that the empirical studies in question do not even try to correlate prices and values and remain at the level of "correlating prices with prices".

Let us examine each of these points in turn.

Labour time: socially necessary and by the clock

The possible discrepancy between Marx's socially necessary abstract labour time and the actual hours of labour performed by particular workers in particular firms, as can be measured directly by the clock, has two aspects. The first concerns variation in the productivity of labour (of a given level of skill) across enterprises, and the second concerns the question of skilled versus unskilled labour.

Suppose that in a given industry some firms use relatively advanced technology and/or organize the labour process in a relatively efficient manner and/or enforce a relatively high intensity of labour, while others fall short on one or more of these criteria. Then the former firms will produce a given product with a lesser, and the latter firms with a greater, input of clock-hours of labour time. Since the actual hours of labour performed in the more efficient and the less efficient enterprises cannot both be "socially necessary" this establishes the point that clock hours are in general not equivalent to socially necessary hours.

But how is this relevant to the calculation of labour-values at industry level? In Marx's conception, the labour time that is socially necessary for the production of a given product is the "normal" or "average" figure. Marx speaks of "the labour time which is necessary on an average, or in other words is socially necessary", and continues thus:

> Socially necessary labour-time is the labour-time required to produce any use-value under the conditions of production normal for a given society and with the average degree of skill and intensity of labour prevalent in that society (Marx, 1976).

It follows that if our data cover the whole industry, the aggregated clock hours will correspond closely to the socially necessary hours, since the total is just the average multiplied by the sample size.

Beyond this arithmetical truism, it also seems clear that Marx envisaged a mechanism that enforces (albeit imperfectly) convergence of the actual hours of labour performed within the various enterprises on the socially necessary average. The idea is that if some firms require a substantially greater than average labour input they will suffer higher costs and will be driven out of business, while those firms that manage to produce a given output with substantially lower than average labour hours will garner super-profits and hence attract emulators. The degree of enforced convergence will be greater, the greater the extent to which the cost-cutting and profit-seeking "logic" of capitalism dominates the market.

From this perspective, the notion that Marx's socially necessary labour time is an in principle unobservable magnitude is just obscurantism. True, there exists no chronometer that measures the socially necessary component of the labour hours performed by a particular worker in a particular firm. But socially necessary labour is indeed measurable statistically; if this were not so, Marx's theory would be a nonsense.

What about the issue of skilled versus unskilled labour? Marx, following Smith and Ricardo, thought of unskilled or "simple" labour as the baseline, with skilled labour conceived as some sort of multiple of simple labour. In the classical conception skilled labour "creates value" at a rate in excess of simple labour —and the excess can be measured by the wage differential. There is a strong whiff of circularity in this notion; it appears to rule out the seemingly meaningful question of whether wage differentials correctly reflect differential rates of "value creation".

Elsewhere we have offered an analysis of this question which breaks the circularity while maintaining the basics of Marx's analysis.[3] In our view, all labour "creates value" (*i.e.* costs an expenditure of finite human time) at a uniform rate but skilled labour also transfers to the product a portion of a previously accumulated sum of labour time, insofar as the skills in question are acquired via the work of the individual and his or her instructors in a definite training process. This analysis follows the model of Marx's analysis of the transfer of the value of means production to the product: acquired human skills "depreciate into" the product much as machines do. It turns out that the implications of this point are best pursued under the next heading.

Wage bills versus labour hours

We now turn to the claim that the authors of price-value correlation studies have not even used clock hours of labour time on the labour-values side of the relationship.

While it true that many studies have used industry wage bills as a proxy for direct labour input, the claim that all empirical studies of price-value correlations have relied on wage data is false. The Swedish input-output tables give labour inputs not in money but in person years. David Zachariah has analysed the price-value relationship for several economies, and he finds that the Swedish data show the same strong correlation that has been observed in other studies (Zachariah, 2006).

In addition, even if an input-output table expresses the labour input in terms of wages it is possible to work backwards to hours if industry-average wage

[3] See chapter 2, and in particular the Appendix on the skilled labour multiplier, of Cockshott and Cottrell (1993).

data are available: one deflates the wage-bill numbers using the sector-specific average wage. Cockshott, Cottrell and Michaelson (1995) did this in their analysis of the United Kingdom economy, obtaining average hourly wage rates per industry from the New Earnings Survey. The result: the correlation between prices and labour-values remained almost as strong as when the wage bill was used as a proxy for sectoral labour input. Specifically, the authors found a price-value correlation of 0.98 when using straight wage-bill data versus 0.96 after adjusting for industry-specific wage rates.

But let us pause on this point for a moment. It's a legitimate question whether the measured person hours expended in a particular industry or the wage bill paid by that industry is the more appropriate measure of the socially necessary labour time devoted to that industry (before considering the indirect labour contributed via the non-labour inputs).

If the variation in the average wage across industries is just "noise" in relation to value analysis then we are better using actual person years data when available, or deflating industry wage bills by industry wage rates, if possible, to arrive at figures in person years. The possibility remains, however, that differences in average wages across industries (partly) reflect differences in the skill composition of the labour force in those industries, and that (as mentioned in the previous section) these differences are associated with differences in the indirect labour contribution due to the education and training of skilled workers. Without further research we cannot say anything very definite on this matter but it seems plausible, at least, that the theoretically "correct" labour input figure may lie between measured person hours and the figure implied by the sector's relative wage bill. But if these two figures support very similar values of the price-value correlation then, of course, the point is moot.

Indirect labour and technical coefficients

The fact that inter-industry coefficients are given as ratios of monetary magnitudes, rather than as ratios of in-kind product flows, is a result of the degree of aggregation of the actually available input-output tables for capitalist economies. In order to construct a meaningful input-output table in natura the data must be fully disaggregated by product, but many of the industries as defined in the actual tables produce a wide range of different products. For example, there can be no meaningful in-kind number for the quantity of output of "aircraft and parts" or "electronic components and accessories", or for the in-kind flow of the product of the latter industry into the former. In a planned economy it would be possible to construct material flows in terms of unique identifiers for each type of product, using bar codes for instance. Since this information is not available to national statistical offices in capitalist economies the practical solution is to present the aggregate monetary values of flows between sectors.

But this does not create a problem, if one is interested in comparing the aggregate monetary value of the output of the industries with the aggregate labour-value of those same outputs, since the vector of aggregate sectoral labour values calculated from a monetary table will agree with the vector calculated from a physical table, up to a scalar, regardless of the price vector and the (common) wage rate used in constructing the monetary table. Or in other words, the vector of sectoral labour values obtained is independent of the price vector used. One might just as well (if it were practically possible) use an arbitrary vector of accounting prices or weights to construct the "monetary" table. The fact that actual prices are used in the published data does not in any way "contaminate" the value figures one obtains; no spurious goodness of fit between values and prices is induced. We provide a proof of this assertion in the Appendix to this paper.

Correlation coefficients between two vectors do not change under scalar multiplication of one of the vectors. A correlation between rainfall and temperature for example is not affected by whether temperature is measured in Fahrenheit or Centigrade, nor whether the rainfall is measured in centimeters or inches. Thus since the aggregate sectoral values obtained from the monetary data agree, up to a scalar, with those that would be obtained from the data in natura, it follows that the correlation coefficients obtained in this way will be the same as those that would be obtained with in-kind data. The sole source of variation would be the assumption that wage rate was the same across industries. But as noted above, tests have already been performed in which correction is made for differing wage rates across industries and the correlation remains very strong.

Spurious correlation

In this section we address the claim of Nitzan and Bichler (2009) that the price-value correlations we and others have obtained are essentially spurious since they do not take into account industry scale. This is a continuation of the argument advanced by Kliman (2002).

We respond to this strand of their argument in three ways:

1. By explaining the hypothesis that is under test in our studies. A clear understanding of this hypothesis will, we believe, undercut the temptation to think of the results as an instance of spurious correlation.
2. By showing that Nitzan and Bichler's demonstration of spurious correlation is based on what in computer science is called a type error, and in physics a dimensional error. This argument we owe to Valle Baeza (2010) and Fröhlich (2010a).
3. By citing additional empirical evidence in support of the idea that the observed correlations between labour content and monetary value are not spurious.

The basic hypothesis being tested

There is a certain irony in Nitzan and Bichler's opposition to the labour theory of value. The idea on which they found their own work —that capital is power— has a respectable classical pedigree; Adam Smith long ago wrote that monetary wealth was power. But Smith was specific: monetary wealth was the power to command the labour of others. If Nitzan and Bichler were to seek a measurable correlate to power they would do well to follow Smith. But in doing so they would have to abandon their opposition to the labour theory of value: power is a power over labour and there is a direct correlation between amounts of money and the amount of labour commanded. This correlation Nitzan and Bichler deny, thus depriving their theory of the realistic foundation that Adam Smith had.

The purpose of the empirical studies that we and others have done on the price-value relationship has been to verify this basic proposition of classical political economy, that labour is the source of commodities' exchange value. We first took up this issue because we wanted to analyse national income in terms of Marxian categories —rate of surplus value, organic composition of capital, and so on. Initially our work was turned down by reviewers on the grounds that we were using monetary quantities to measure what should have been labour-value ratios. In order to establish the validity of our procedure we showed that even if you break the economy down in much finer detail, using input-output tables, there was a close correlation between labour-value magnitudes and monetary magnitudes. It was therefore valid to use monetary data to work out ratios such as the organic composition of capital or the rate of exploitation.

Marx's analysis of capitalist exploitation rests on the hypothesis that embodied labour —or to put it more exactly, concurrently required labour— is the source of monetary value. To establish the validity of this hypothesis and the analysis of exploitation that stems from it, it is sufficient to break down the economy into a large number of sectors and show that the monetary value of the gross output of these sectors correlates closely with the labour concurrently expended to produce that gross output. This in turn requires that you compute two vectors.

1. A vector of monetary flows of output indexed by sector, each element of which is of dimension currency-units per year.
2. A vector of the number of people whose annual labour is directly or indirectly embodied in this monetary output, the dimension of each element of which is a number of persons, since person hours per annum reduces to the dimension persons. This, incidentally is exactly the format used by the Swedish input-output tables mentioned above.

If a strong correlation exists between the two vectors we can say that the data are consistent with the hypothesis that labour is the source of value. It must be emphasised that this method directly examines what we want to test, namely, whether monetary value is proportional to labour used.

The argument that the correlations observed are spurious depends on the idea that there exists an independent third factor that is the cause of concomitant variation in the persons and monetary flow vectors. Any correlation observed in science could potentially be spurious, so this is always a possibility. But for an allegation of spurious correlation to be borne out, one must both identify this third factor and show that it actually does induce the correlations observed. So what could this third factor be?

Kliman (2002) suggests that it is industry size. "Big" industries employ more people and also sell more output, and the correlation between sectoral prices and values arises just because of this fact. But for a third factor to be the common cause of the variation in the two vectors of interest, that factor must itself be quantifiable. How do you measure industry size? The most obvious measures of an industry's size —how many people it employs, or its turnover— are ruled out, since we are looking for something independent. Kliman, Bichler and Nitzan suggest that there is some third form of industry size that causes the variations in both employment and turnover.

There certainly are other possible measures, for example the area of land an industry occupies, the number of tons of output it produces, or the number of megawatt-hours of energy it uses. In principle any of these could be the third factor that determines both the labour used and the turnover of an industry, but we merely have to list them to see how implausible it is that land area or tonnage is an appropriate third source of variation.[3]

Agriculture is by far the "largest" industry in the United Kingdom in physical terms. It occupies the most space, but its employment and turnover are in no way proportional to its size in these terms. The water supply industry is the largest in terms of kilograms delivered, but again, its position in terms of turnover and employment falls far short.[4]

In fact, however, Bichler and Nitzan don't propose any of the measures of size mentioned above; instead they suggest that the common cause of variation is simply the number of units of output produced. They produce a spreadsheet showing that if you take two uncorrelated vectors (**a** and **b**) and multiply them, element by element, by a third random vector (**c**), then the result **a** o **c** will be correlated with **b** o **c**. Mathematically this is fine, but it has no relevance to the question under dispute unless some economic meaning can be given to the vectors **a**, **b** and **c**. Putting headings at the top of the columns such as "unit price" and "number of units sold" does not give their example any grip unless they can explain what these "units" are in the context of input-output tables.

So if we look at the United Kingdom industrial sectors what are the units of output? For industry 30 —footware—, it is presumably pairs of shoes. But what is the unit of output for industry 47 —rubber products—, or industry 50 —ceramic goods—, or industry 67 —weapons and ammunition—, is the unit of output a bullet, a tank or an atom bomb? In electricity production, is the unit the kilowatt-hour, the megawatt-hour or what? In milk production, the pint, gallon or litre?

There are two issues here. First, many industries defined at the level of detail of national input-output tables produce heterogeneous output for which there exists no common unit; and second, many industries produce "bulk" output for which the scale of the unit is arbitrary.

4 Energy input is a more plausible third factor. We return to this point in the last subsection of this section.

Niztan and Bichler write that "most people" think of price as an attribute of "individual commodities", as in "the price of a Toyota Corolla, the price of a bushel of wheat, the price of a United Airlines flight from New York to Tokyo" (2009: 95). That may be so —and certainly there are cases where the relevant unit seems obvious— but then "most people" have not thought about how to measure the economy-wide relationship between price and value and have no sense of the conundrums that arise if you try to define "units" for all the goods that are produced.

An impossible correlation

We have argued above against the idea that one can come up with a well-defined set of natural "units" for the output of each sector of the economy, such that one could analyse the relationship between prices per unit and labour-value per unit. In this section we suspend disbelief in such units: for the sake of argument we suppose that there is a definite natural unit for each commodity. Hence there is a well-defined vector of prices ($\mathbf{p}$), where $\mathbf{p}_i$ is the price of industry i's product per natural unit); a corresponding well-defined vector of per-unit values ($\mathbf{v}$) and a well-defined vector of industry sizes ($\mathbf{q}_i$); $i = 1,\ldots,n$, expressed in terms of natural units.

The point pressed by Nitzan and Bichler is that the "proper" correlation to consider is that between price-per-unit and value-per-unit, while the correlation actually examined by those claiming to test the labour theory of value is that between aggregate sectoral prices ($\mathbf{p}_i\mathbf{q}_i$) and aggregate sectoral values ($\mathbf{v}_i\mathbf{q}_i$) —and multiplication by qi induces a spurious correlation. As mentioned above, they claim to illustrate this point using a spreadsheet in which the original $\mathbf{p}$,$\mathbf{v}$ correlation is minimal but, due to variation in industry size, the correlation of $\mathbf{p}_i\mathbf{q}_i$ and $\mathbf{v}_i\mathbf{q}_i$ is substantial.[5]

Our response here is that while the correlation of $\mathbf{p}_i\mathbf{q}_i$ and $\mathbf{v}_i\mathbf{q}_i$ is mathematically valid (and not spurious, as we have argued above), the putative correlation of $\mathbf{p}_i$ and $\mathbf{v}_i$ is mathematically invalid and not meaningful.

When Nitzan and Bichler calculate their initial $\mathbf{p}$,$\mathbf{v}$ correlation they rely on the CORREL function built into Microsoft Excel. Now Excel is in computer

5 See: <http://bnarchives.yorku.ca/308/04/20101200_cockshott_nitzan_bichler_testing_the_ltv_spurious_correlati on.xls>.

science terms an "untyped" program. It does not check that the mathematical operations one is performing make sense since it knows nothing about what the numbers in a spreadsheet represent. More rigorous programming systems like Fortress (Allen *et al.*, 2005) or Vector Pascal (Cockshott, 2002) allow the user to specify the units being used for variables so that dimensional analysis can be applied. Had this been done the computer would have warned Bichler and Nitzan of the mistake they were making.

Dimensional analysis is a set of rules to verify basic aspects of mathematical models; it specifies necessary (but not sufficient) conditions for the validity of a model. Variables, in general, are ordered pairs —a magnitude x and a unit of measurement $[m]$— for example the oil price might be 90 [$/barrel]. The basic rules of dimensional analysis are as follows.[6]

1. Any mathematical expression must be dimensionally consistent, that is, the units on the left-hand side of the expression must be the same as the units on the righthand side.
2. Addition or subtraction of magnitudes with same units is allowed: $x[m] + y[m] = (x + y)[m]$.
3. Addition or subtraction of magnitudes with different units is not allowed: $x[m] + y[t]$ is impossible.
4. Multiplication of variables with different units is allowed: $x[m]y[t] = (xy)[mt]$.
5. Division of variables with different units is also acceptable: $x[m]/y[t] = (x/y)[m/t]$.

In dimensional terms the unit value of commodity i is $\mathbf{v}_i[h/u_i]$ and its unit price is $\mathbf{p}_i[\$/u_i]$ where h denotes labor time and ui the appropriate physical unit of product i. Now the correlation coefficient between two vectors $\mathbf{x}$ and $\mathbf{y}$ is the inner product of the normalised vectors:

$$\rho(\mathbf{x}, \mathbf{y}) = N(\mathbf{x}).N(\mathbf{y})$$

where the normalisation function $N(x)$ for a vector $\mathbf{x}$ subtracts its mean $(\mu_\mathbf{x})$ and divides by its standard deviation $\sigma_\mathbf{x}$:

$$N(\mathbf{x}) = (\mathbf{x} - \mu_\mathbf{x})/\sigma_\mathbf{x}$$

The correlation between per-unit price and per-unit value would therefore be given by the expression:

[6] For a fuller account see Fröhlich (2010a), and de Jong and Quade (1967).

$$\rho(\mathbf{p},\mathbf{v}) = N(\mathbf{p}).N(\mathbf{v}) = \left(\frac{\mathbf{p}-\mu_{\mathbf{p}}}{\sigma_{\mathbf{p}}}\right)\cdot\left(\frac{\mathbf{v}-\mu_{\mathbf{v}}}{\sigma_{\mathbf{v}}}\right)$$

So what's the problem here? To find $\mu_{\mathbf{p}}$ we have to sum the unit prices of the various commodities, and to find $\mu_{\mathbf{v}}$ we have to sum their unit values. But this operation is forbidden by rule 3 above. For example the addition of the price of oil $\mathbf{p}_o$ [$/barrel], and the price of a pencil $\mathbf{p_p}$ [$/pencil], is ruled out, since it involves incompatible dimensions. (Besides the rules, intuitively there's clearly something wrong with the notion of the average price of a barrel of oil and a pencil, as opposed to, say, the average price of oil across different markets or periods.) Since normalisation depends on computing the mean of a vector, and computing the mean depends on addition, normalisation is only defined on vectors of homogeneous dimension. And so correlation likewise only applies to vectors of homogeneous dimension.

The attempt to compute $\rho(\mathbf{p},\mathbf{v})$ in per-unit terms fails because the vectors $\mathbf{p}$ and $\mathbf{v}$ are not dimensionally homogeneous; they represent not n values of two variables but rather two arrays each holding n different variables. On the other hand the correlation of prices multiplied by quantities $\mathbf{p}_i\mathbf{q}_i$, and labour-values multiplied by quantities $\mathbf{v}_i\mathbf{q}_i$, is well defined because each $\mathbf{p}_i\mathbf{q}_i$ has dimension [$] and each $\mathbf{v}_i\mathbf{q}_i$ has dimension $[h]$. Each is of homogeneous dimension and thus correlation is well defined on them.

Proponents of the spurious correlation criticism have confused the problem: there is no original correlation of two variables with n observations complicated by introducing a third variable. The allegedly spurious correlation is a meaningful way to measure the price-value relationship and Nitzan and Bichler's supposedly non-spurious correlation is a total mistake.

Some real research questions

The spurious correlation argument turns out to be a red herring. Nonetheless, there are meaningful ways of addressing the suspicion that lies behind that argument. In this section we briefly discuss two avenues of empirical research that can additional shed light on the matter.

First, the labour-value of a commodity or set of commodities represents the sum of the direct and indirect labour time required for its production. In the terminology used by Pasinetti (1981), labour-values may be described as vertically integrated labour coefficients. In a similar manner it is possible to calculate (from the same input-output data) vertically integrated coefficients for selected inputs other than labour. For such a calculation to be economically meaningful the selected input must be reasonably homogeneous; possible candidates might include oil, electricity and steel. That is, one can define "oil-values", for example, in a fashion analogous to labour-values. The question then arises: how does the correlation between sectoral prices and oil-values compare with that between prices and labour-values? Note that if such correlations were simply an artifact of differential industry "size" (however measured), one would expect to find a similar correlation for any selected input.

This sort of analysis was done by Cockshott and Cottrell (1997a). Using United Kingdom input-output data we tested the candidate "value bases" oil, electricity, and iron and steel, and found correlations against price of 0.799, 0.826 and 0.576 respectively, as compared with 0.977 for labour.[7] For the record, we don't consider any of these correlations to be spurious, but the point is that labour-values produce a much closer fit to sectoral prices than the alternatives.

Second, we have concentrated above on the correlation coefficient as the statistic by means of which to assess the price-value relationship. But correlation is not the only relevant tool —we focused on it largely because we're concerned to rebut the charge of spurious correlation— and it is worth mentioning an important alternative approach. That is, one can focus on the ratio of aggregate price to labour-content (or alternatively labour-content to price) across the sectors of the economy. If the labour theory of value is correct, these ratios should be fairly narrowly distributed. (If the theory held exactly, which of course we do not expect to find, all the sectoral price-value ratios would be identical.) This approach was suggested at a theoretical level by Farjoun and Machover (1983) and was pursued empirically by Cockshott and Cottrell (1998). The appropriate summary statistic here is the coefficient of variation or CV (that is, the ratio of the standard deviation to the mean).[8] The idea is that

7 See Tables 1 and 2 in Cockshott and Cottrell (1997a). The goodness of fit statistics given in the tables are R^2 values; here we have taken their square roots to obtain correlation coefficients since we're talking about correlation.

8 In a passing nod to the idea that incomparable units are a problem, Niztan and Bichler suggest that to circumvent the problem one might, for example, "correlate the ratio between the price of cereals and the price of aircraft on the one hand with the ratio between the value of cereal and the value of aircraft on the other" (2009: 95, n7). This appears to be a somewhat garbled version of what we're discussing.

if prices are "close" to labour-values the CV of the price to value ratio across sectors should be "small".

One difficulty with this approach is that the CV is not a standardized statistic (unlike the correlation coefficient, which satisfies $-1 \leq \rho \leq 1$), so it is not immediately obvious what is "small". But one can compare the coefficient of variation across different candidate value bases as discussed above. Looking at the ratio of x-content to price for 100 sectors of the United Kingdom economy, Cockshott and Cottrell (1997a) found CVs of 0.198 for x = labour, 11.41 for oil, 3.69 for electricity and 7.81 for iron and steel; the labour-based figure is clearly much smaller than the others.

A related line of research would be to compare the CV of price-value ratios using input-output tables of different degrees of disaggregation. Most national input-output tables comprise around 50 to 100 sectors; the US Bureau of Economic Analysis, however, has made available a table with over 400 sectors. One might expect to find a somewhat broader dispersion of price-value ratios when using more disaggregated data, since less mixing and averaging would be going on. Just how much difference this would make is an empirical question to which we do not yet have an answer. Let us emphasize, this sort of research would give us useful additional information on the price-value relationship —on how robust it is at a more "micro" level— in contrast to the sterile and confused charge of spurious correlation.

Conclusion

We have considered two charges laid by Nitzan and Bichler against the research that claims to provide empirical support for the labour theory of value.

Against their objection that the research in question fails to uncover labour-values, and remains within a circle of price-price relationships, we have countered that sectoral wage bills give a reasonable proxy for Marx's socially necessary labour time and moreover that labour-time figures are available for some countries and have been used in the literature, while it is also possible to back out labour-time data from wage bills given average wage-rate data by industry. On this point we have also argued that having to use inter-industry coefficients calculated as ratios of monetary magnitudes, rather than as ratios of in-kind flows, does not in fact compromise the analysis.

Against the charge of spurious correlation we have presented an array of arguments to undercut the idea that there exists a "correct" (but possibly null) correlation between prices and values at the level of the individual commodity, which is artificially inflated by the use of industry-level data. The supposedly correct correlation is in fact invalid, breaking the rules of dimensional analysis, while a good deal of ancillary evidence supports the validity of the finding of a close relationship between prices and values —a finding which can also be expressed without recourse to correlation. Interesting open questions remain regarding the price-value relationship, but whether it's a case of spurious correlation is not one of them.

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Appendix: proof of invariance of measured values

Here we prove that sectoral labour-values calculated on the basis of a "monetary" input-output table of the sort provided by national statistical offices are invariant in this sense: they do not depend on the prices used in forming the matrix of technical coefficients, where these coefficients are expressed in terms of the dollars-worth of input from industry j required to produce a dollar's worth of output in industry i. For the purposes of this argument we assume a common wage rate ($\mathbf{w}$) across all industries. This argument was originally presented in Cockshott and Cottrell (1997b).

Consider an economy characterized by the following arrays:

- U An $n \times n$ matrix of inter-sectoral product ows in kind, such that u_{ij} represents the amount of industry j's output used as input in industry i.
- $\mathbf{q}$ An $n \times 1$ vector of gross outputs of the industries, in their natural units.
- $\mathbf{l}$ An $n \times 1$ vector of direct labour-hours performed in each industry.

It will be useful also to define an $n \times n$ diagonal matrix Q such that:

$$Q_{ij} = \begin{cases} \mathbf{q}_i & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

The standard calculation of labour-values proceeds as follows. First calculate the $n \times n$ matrix of technical coefficients as $A = Q^{-1}\,U$ and the $\mathbf{n}$-vector of direct labour input per unit of physical output as $\boldsymbol{\lambda} = Q^{-1}\,\mathbf{l}$. The $\mathbf{n}$-vector of unit values (vertically integrated labour coefficients) is then given by:

$$\mathbf{v} = (I - Q^{-1}\,U)^{-1}Q^{-1}\mathbf{l} = (I - A)^{-1}\boldsymbol{\lambda}$$

and the $\mathbf{n}$-vector of aggregate values of the sectoral outputs is:

$$\mathbf{V} = Q\mathbf{v} = Q(I - A)^{-1}\boldsymbol{\lambda} \qquad [1]$$

We now construct the monetary counterpart to the above arrays. Let the $\mathbf{n}$-vector p represent the prices of the commodities and the scalar $\mathbf{w}$ denote the (common) money wage rate. Let us also define an $n \times n$ diagonal matrix P such that:

$$P_{ij} = \begin{cases} \hat{\mathbf{q}}_i & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} = QP$$

Corresponding to each of the initial "real" arrays above there is a monetary version as follows:

$\hat{U} = UP$	Matrix of money-values of inter-sectoral product flows.
$\hat{\mathbf{q}} = P\mathbf{q}$	Vector of money-values of gross outputs.
$\hat{\mathbf{l}} = \mathbf{wl}$	Vector of industry wage-bills.

From these we can construct counterparts to the derived "real" arrays. First the $n \times n$ diagonal matrix $\hat{Q}$, whose diagonal elements are $\mathbf{p}_i\mathbf{q}_i$, is given by:

$$\hat{Q}_{ij} = \begin{cases} \hat{\mathbf{q}}_i & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} = QP \qquad [2]$$

The counterpart to the matrix of technical coefficients is:

$$\hat{A} = \hat{Q}^{-1}\hat{U} = (QP)^{-1}UP = P^{-1}Q^{-1}UP = P^{-1}AP \qquad [3]$$

The elements of $\hat{A}$ represent the dollars' worth of input from sector j required to produce a dollar's worth of output in sector i. Finally, the counterpart to $\boldsymbol{\lambda}$ is the $\mathbf{n}$-vector $\boldsymbol{\lambda}$.

$$\hat{\boldsymbol{\lambda}} = \hat{Q}^{-1}\hat{\mathbf{l}} = (QP)^{-1}\mathbf{wl} = \mathbf{w}P^{-1}Q^{-1}\mathbf{l} = P^{-1}\mathbf{w}\boldsymbol{\lambda} \qquad [4]$$

whose elements represent the direct labour cost per dollar's worth of output in each sector.

Now here is the issue: suppose we are not privy to the information on product flows in kind and labour-hours, and have at our disposal only the information given in the monetary tables. On this basis we can calculate a vector $\hat{\mathbf{v}}$:

$$\hat{\mathbf{v}} = (I - A)^{-1}\boldsymbol{\lambda}$$

While $\mathbf{v}_i$ represented the vertically integrated labour hours per physical unit of output of commodity i, the $\hat{\mathbf{v}}_i$ that we are able to obtain from the monetary tables represents the vertically integrated labour cost per dollar's worth of output of commodity i. If we then multiply up by the money-value of the gross outputs of the industries we obtain the vector of vertically integrated labour costs for the industries.

$$\hat{\mathbf{V}} = \hat{Q}\hat{\mathbf{v}} = \hat{Q}(I - \hat{A})^{-1}\hat{\boldsymbol{\lambda}} \qquad [5]$$

We are interested in the relationship between [1], the aggregate sectoral values that could be obtained in principle from the data in natura, and [5], the corresponding figures obtained by using the monetary data.

On the basis of the correspondences [2], [3] and [4] we can rewrite [5] as:

$$\hat{\mathbf{V}} = QP(I - P^{-1}AP)^{-1}P^{-1}\mathbf{w}\boldsymbol{\lambda} \qquad [6]$$

Recall that [1] specified $\mathbf{V} = Q\,(I - A)^{-1}\boldsymbol{\lambda}$. Comparing these two equations we see that $\hat{\mathbf{V}} = \mathbf{wV}$ on condition that:

$$(I - A)^{-1} = P(I - P^{-1}AP)^{-1}P^{-1} \qquad [7]$$

That this condition is indeed satisfied may be seen by taking inverses on both sides of [7]. On the left, we simply get $(I - A)$; on the right we get:[9]

$$[P(I - P^{-1}AP)^{-1}P^{-1}]^{-1} = P(I - P^{-1}AP)P^{-1} = (P - AP)P^{-1} = I - A$$

This means we have proved that $\hat{\mathbf{V}} = \mathbf{wV}$, which is to say that the aggregate sectoral values obtained from the monetary data agree —up to a scalar, namely $\mathbf{w}$, the common money wage rate— with those that would be obtained from the data in natura, if these were available. The aggregate value vector is independent of the price vector used in forming the monetary tables.

[9] Using repeated application of the rule that $(AB)^{-1} = B^{-1}A^{-1}$ for any non-singular A and B.