PLOS ONE

RESEARCH ARTICLE

Energy and institution size

Blair Fix*

York University, Faculty of Environmental Studies, Toronto, ON, Canada

* blairfix@gmail.com

Abstract

Why do institutions grow? Despite nearly a century of scientific effort, there remains little consensus on this topic. This paper offers a new approach that focuses on energy consumption. A systematic relation exists between institution size and energy consumption per capita: as energy consumption increases, institutions become larger. I hypothesize that this relation results from the interplay between technological scale and human biological limitations. I also show how a simple stochastic model can be used to link energy consumption with firm dynamics.

Check for updates

OPEN ACCESS

Citation: Fix B (2017) Energy and institution size. PLoS ONE 12(2): e0171823. doi:10.1371/journal.pone.0171823

Editor: Frank van Rijnsoever, Utrecht University, NETHERLANDS

Received: September 11, 2016

Accepted: January 26, 2017

Published: February 8, 2017

Data availability statement: All relevant data are within the paper and its Supporting Information files.

Funding: This research was supported by the Social Sciences and Humanities Research Council of Canada (767-2015-2015, sshrc-crsh.gc.ca). The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing interests: The author has declared that no competing interests exist.

1 Introduction

Throughout the last century, there has been a recurrent desire to connect human social evolution to changes in energy consumption [1–4]. The motivation is simple: the laws of thermodynamics dictate that any system that exists far from equilibrium must be supported by a flow of energy [5]. Since human societies are non-equilibrium systems, it follows that energy flows ought play an important part in social evolution. However, it has proved difficult to move from grand pronouncements based on the laws of thermodynamics to a *quantitative* understanding of the relation between energy use and social evolution [6]. This paper offers a contribution to such a quantitative understanding.

This paper is concerned with one particular aspect of social change: the growth in size of the *institutions that control human labor*. While such institutions have taken many forms throughout history, in the modern era, the control of human labor is dominated by two institutions: the *business firm* and *government*. In this paper, institution *size* refers to the amount of human labor (i.e employment) controlled by an organization. Under this definition/metric of institution size, I demonstrate that a pervasive, positive correlation exists between institution size and energy use per capita.

I pursue two avenues for understanding the relation between energy and institution size. The first approach draws on the rich history of stochastic modelling within firm size theory. Stochastic (random) models have been successfully used to link firm *dynamics* to the overall firm size distribution. Yet there is little understanding of what drives variations in firm dynamics. Using data on firm age and firm size to constrain a stochastic model, I demonstrate that firm dynamics are likely related to rates of energy consumption, and I offer a prediction of what this relation should look like.

The second approach is more speculative, and aims to offer a general explanation of why rates of energy consumption are related to institution size. I propose two factors that mediate this relation: *technological scale* and *social hierarchy*. I hypothesize that increases in energy consumption involve a trend towards the use of technologies that are larger and more complex. These increasingly large technologies require the coordination of greater numbers of people. Given the limitations of the human brain [7], I argue that large-scale social coordination is most easily achieved through social *hierarchy* [8] and that firms and government are specific manifestations of this hierarchy.

This paper is organized as follows. After a brief review of the strengths and weaknesses of various theories of institutional size (Sec. 1.1), Section 2 discusses the empirical evidence connecting energy consumption with institution size. Section 3 then uses a stochastic model to further illuminate the relation between energy use and firm dynamics. Finally, Section 4 presents and tests a series of hypotheses linking institution size to technological scale and social hierarchy.

1.1 Theories of institutional size

Theories of institution size can be divided into two classes: those that concern themselves with the *causes* of institutional growth ('why' theories) and those that do not ('how' theories). 'How' theories have met with great empirical success, while 'why' theories have struggled to offer explanations that are testable.

All 'how' theories of institutional size can be traced back to the work of the French economist Robert Gibrat, who discovered that the rate of growth of business firms seemed to be *independent* of their size [9]. While later investigation found this 'law of proportional effect' to be only approximately true—growth rate variance tends to decline with size [10–12]—it has led to a rich history of stochastic firm growth models [13, 14]. The basic principle is that firm growth is treated probabilistically. Each firm is submitted to a series of random shocks that make it grow (or shrink) over time. When applied to large numbers of firms, the result is a firm size *distribution*. The surprising finding is that these purely random models can very accurately predict the functional form of real-world firm size distributions (see S1 Appendix part F).

Despite their success, 'how' theories are not particularly satisfying because they do not explain *why* institutions grow. Unfortunately, theories that *do* attempt to explain the cause of institution growth often rely on unmeasurable variables, and as a result, are untestable.

The theory of the firm has been dominated by Ronald Coase's *transaction cost* approach. According to Coase, ". . . a firm will tend to expand until the costs of organizing an extra transaction within the firm become equal to the costs of carrying out the same transaction by means of an exchange on the open market or the costs of organizing in another firm" [15]. Unfortunately, transaction costs have been notoriously difficult to define (let alone measure), rendering Coasian theory untestable [16, 17]

Other theories propose that management talent is the driver of firm growth. For instance, Robert Lucas assumes that the firm size distribution results from "allocat(ing) productive factors over managers of different ability so as to maximize output" [18]. Yet Lucas concedes that the causal factor in this model—the talent of managers—is "probably unobservable". Despite this problem, Lucas's theory remains popular [19, 20].

Still other theories propose that firm growth is the result of a resource-driven competitive advantage [21, 22]. Unfortunately, this approach has struggled to stipulate exactly how a particular resource is transformed into a value-creating competitive advantage. Priem and Butler argue that the 'resource-based view' advances a theory of value that is tautological—resources create *value* because they are (among other things) *valuable* [23].

In terms of measurability, theories of government size have faired no better than theories of firm size. One approach is to apply the rational-choice model to the behavior of voters. Government size is treated as a reflection of the preferences of utility maximizing voters [24, 25]. However, without an objective measure of individuals' internal preferences, this theory is untestable.

Another approach is to assume that government bureaucracies (or government as a whole) are self-serving entities that attempt to maximize their budgets, but are restrained by voters and/or an institutional framework such as the constitution [26, 27]. While maximizing behavior is one of the fundamental postulates of neoclassical economics, the hypothesis that humans maximize external pay-offs has been falsified [28].

The lack of measurable variables has consistently plagued 'why' theories of institution size. If a new theory is to be successful, it must demonstrate a connection between institution size and some universally measurable quantity. Energy consumption is just such a quantity.

2 Energy and institution size: Empirical evidence

To study the relation between energy and institution size, I compare variations in energy use per capita to variations in the size of firms and government over both space and time. For firms, I investigate how changes in the base, tail and mean of the firm size distribution are related to changes in energy use per capita. I use self-employment data to investigate the base of the firm size distribution (relying on the assumption that self-employer firms are very small). To investigate the tail of the firm size distribution, I look at the employment share of the largest firms. To quantify the relative size of government, I measure the government share of total employment.

Comparison of these institution size metrics with energy use per capita are shown in Figs 1–3. International trends are shown in Fig 1 (each colored line represents the path through time of a specific country), while Fig 2 shows time-series data for United States. In Fig 3, I focus only on firms and merge data from Figs 1 and 2 and add US *sectoral* and *subsectoral* level data. Although this synthesis merges data that are not identically defined (see Fig 3 caption), the result is clear: the inclusion of sectoral data serves to extend (by two orders of magnitude) the trends found at the national level. In the case of small firms and mean firm size, the inclusion of sectoral data also increases the regression strength.

To summarize our findings, the evidence in Figs 1–3 suggests the following 'stylized' facts. As energy use per capita increases:

1. The small firm employment share *declines*;
2. The large firm employment share *increases*;
3. The mean firm size *increases*;
4. The government employment share *increases*.

Findings 1–3 suggest that increases in energy consumption are associated with a shift in employment from small to large firms. This indicates that the firm size distribution becomes more *skewed* as energy consumption increases. In S1 Appendix (part C), I demonstrate that this shift (at the national level) can be accurately modelled in terms of the changing exponent of a power law distribution.

Assuming a correlation between energy use and GDP, then the evidence presented here is consistent with previous research that has focused on the relation between firm size and GDP per capita [18, 20, 29–31]. However, my focus here on energy use (rather than GDP) is intentional: it is part of a larger effort to ground economic theory in the laws of thermodynamics [32], and to root empirical analysis in biophysical (rather than monetary) phenomena [33–36].

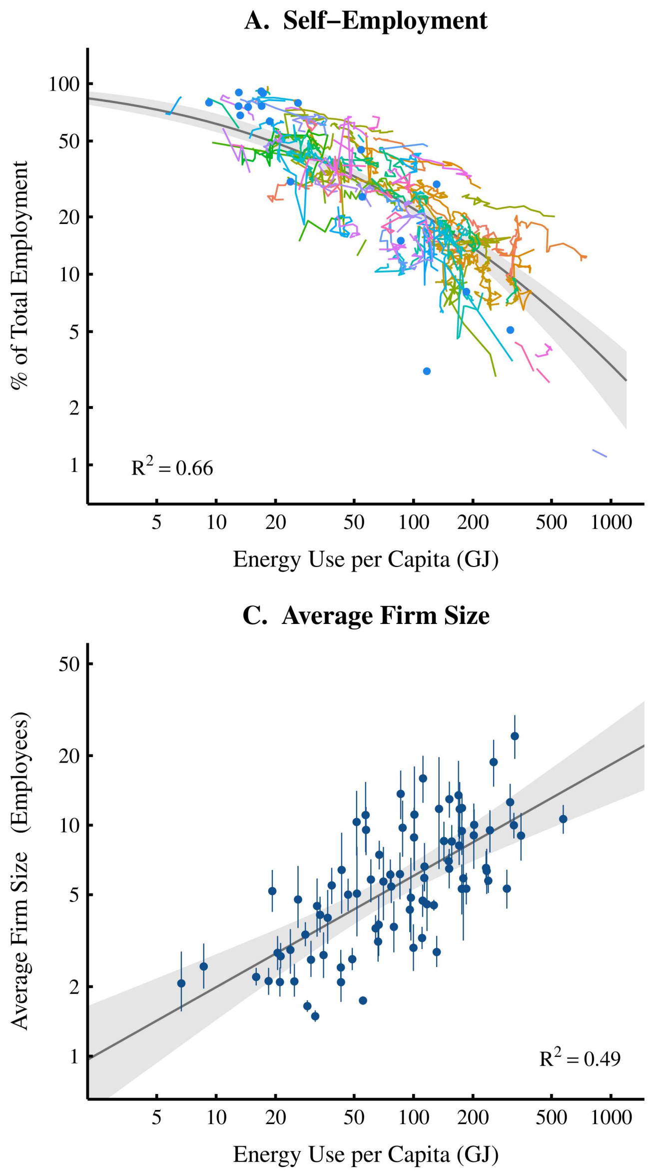

Fig 1. Institution size vs. energy use per capita at the international level. This figure shows how different metrics of institution size vary with energy consumption per capita. Panels A-C analyze variations in firm size by looking at the base, tail, and estimated mean of the firm size distribution. Panel D analyzes variations in government size. In order to show as much evidence as possible, panels A, B and D are a mix of time series and scatter plot. Lines represent the path through time of individual countries while points represent a country with a single observation. Error bars in panel C represent the 95% confidence interval of mean firm size estimates. Variations in self-employment, large-firm, and government employment share vs. energy are modelled with log-normal cumulative distribution functions. Mean firm size vs. energy is modelled with a power law. Grey regions indicate the 99% confidence region of each model. For sources and methodology, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.g001

Following the long-standing division in institution size theory between 'how' and 'why' theories, I adopt two separate approaches for understanding the relation between institution size and energy consumption. The first approach deals with the 'how' question: *how* exactly do changes in firm size occur? To answer this question, I use a stochastic model to illuminate the relation between energy use and firm dynamics. The second approach deals with the more difficult 'why' question: *why* is institution size related to energy consumption. To answer this question, I investigate the relation between energy, technological change, and social coordination.

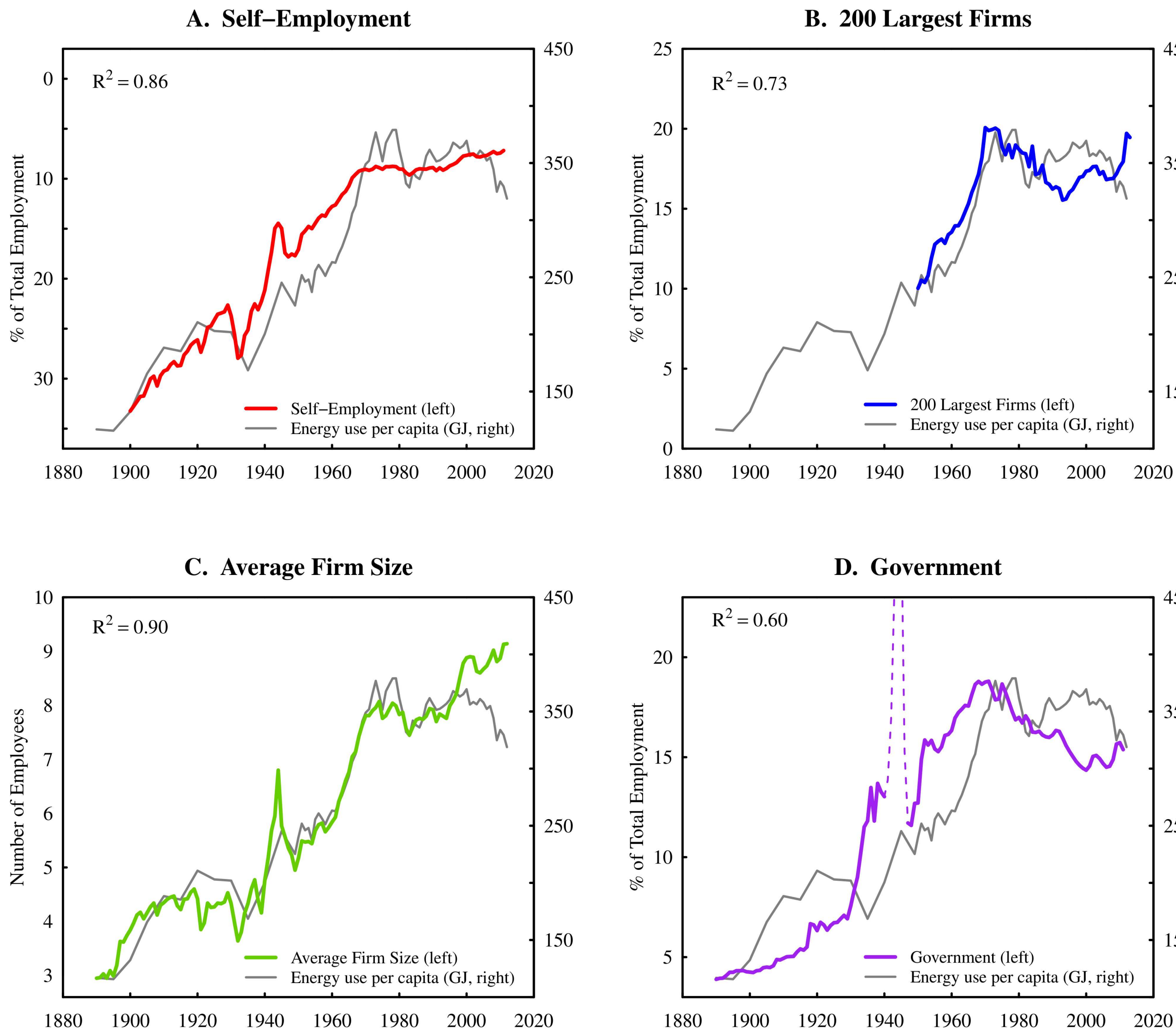

Fig 2. Institution size vs. energy use per capita in the United States. This figure shows the trends for various measures of institution size in the United States over the last century. Trends mirror those found at the global level. As energy consumption per capita increases, self-employment rates decline (panel A, note reverse scale), the large firm employment share increases (panel B), mean firm size increases (panel C), and the government employment share increases (panel D). Note that government regressions exclude World War II (dotted line). For sources and methodology, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.g002

3 The 'how' question: Energy and firm dynamics

Beginning with the work of Gibrat [9] and later Simon and Bonini [37], stochastic models have been successfully used to explain the functional form of the firm size distribution in terms of firm *dynamics*. The implication of these models is that changes in average firm size occur through changes in firm dynamics. Given the connection between energy consumption and firm size, it follows that firm dynamics ought to vary with changes in energy consumption.

Ideally, we would look at this relation directly by investigating international variations in the firm growth rate distribution and comparing them to variations in energy consumption. Unfortunately, data constraints make such a comparison difficult. Calculating international firm growth rate distributions would require longitudinal data for a large, representative

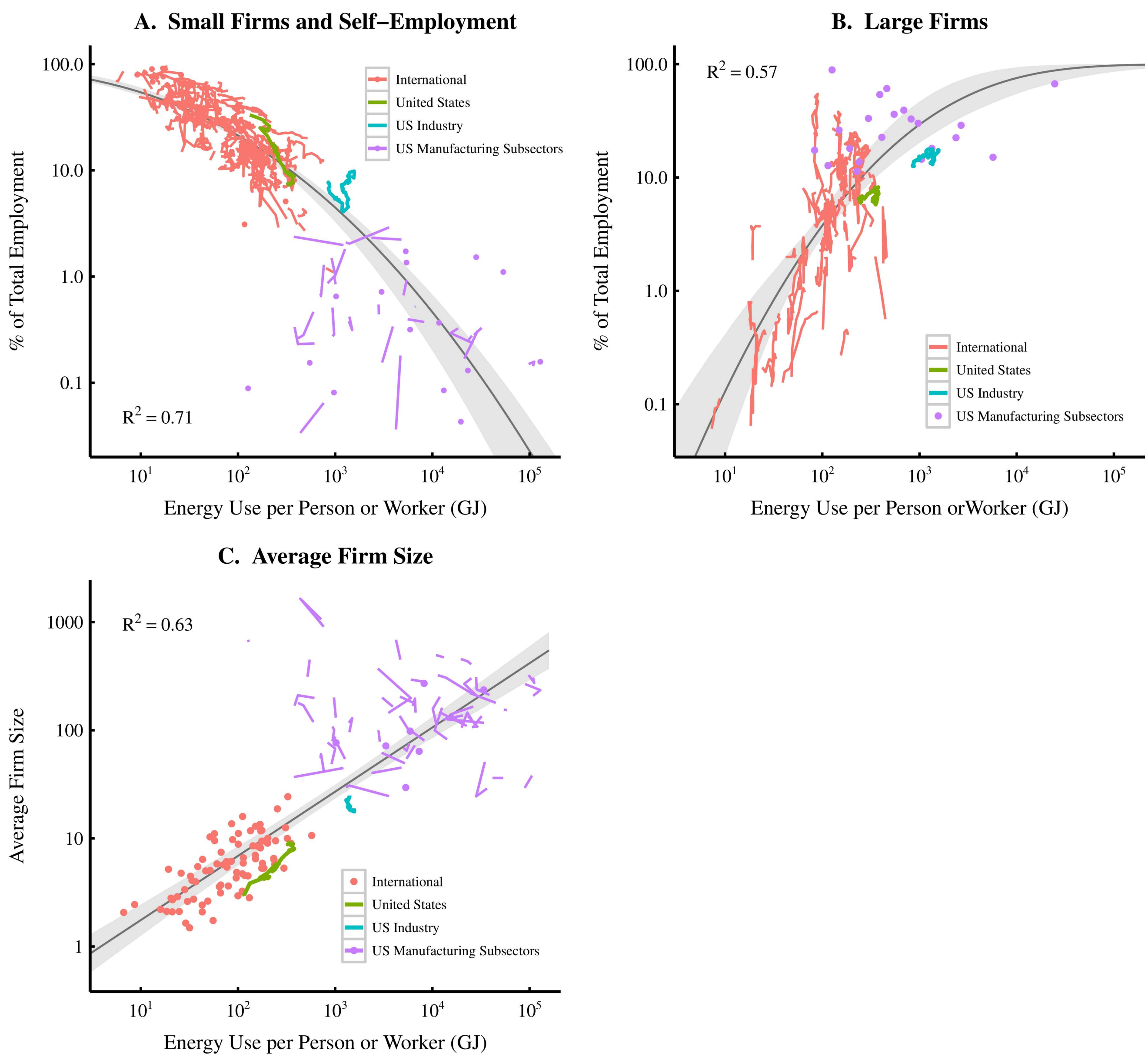

Fig 3. Synthesizing evidence—firm size vs. energy use per person/worker. This figure combines data from 3 different units of analysis (nations, sectors, and sub-sectors) to offer a comprehensive picture of the relation between firm size and energy use per capita (or per worker). 'US Industry' consists of construction and manufacturing sectors, while 'US Manufacturing Subsectors' are the smallest subdivisions of the manufacturing sector. At the national level, energy use is measured per *person*, while at the sectoral level, it is measured per *worker*. In panel A, self-employment data (for nations and US Industry) is merged with the data for the employment share of firms with 0–4 employees in US manufacturing subsectors. In panel B, data for the employment share of the largest 25 firms (for nations and US Industry) is merged with data for the employment share of firms with more than 5000 employees in US manufacturing subsectors. Panel C shows mean firm size data at the national and sectoral level. Grey regions indicate the 99% confidence region of each model. For sources and methodology, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.g003

sample of firms in many countries. I am not aware of the existence of any such data at the present time. However, we can use what little data *is* available to make inferences about the relation between energy and firm dynamics.

Firm age data provides an indirect window into firm dynamics. If we assume that new firms start at a small size, then we can infer the historic rate of growth of any firm, given its

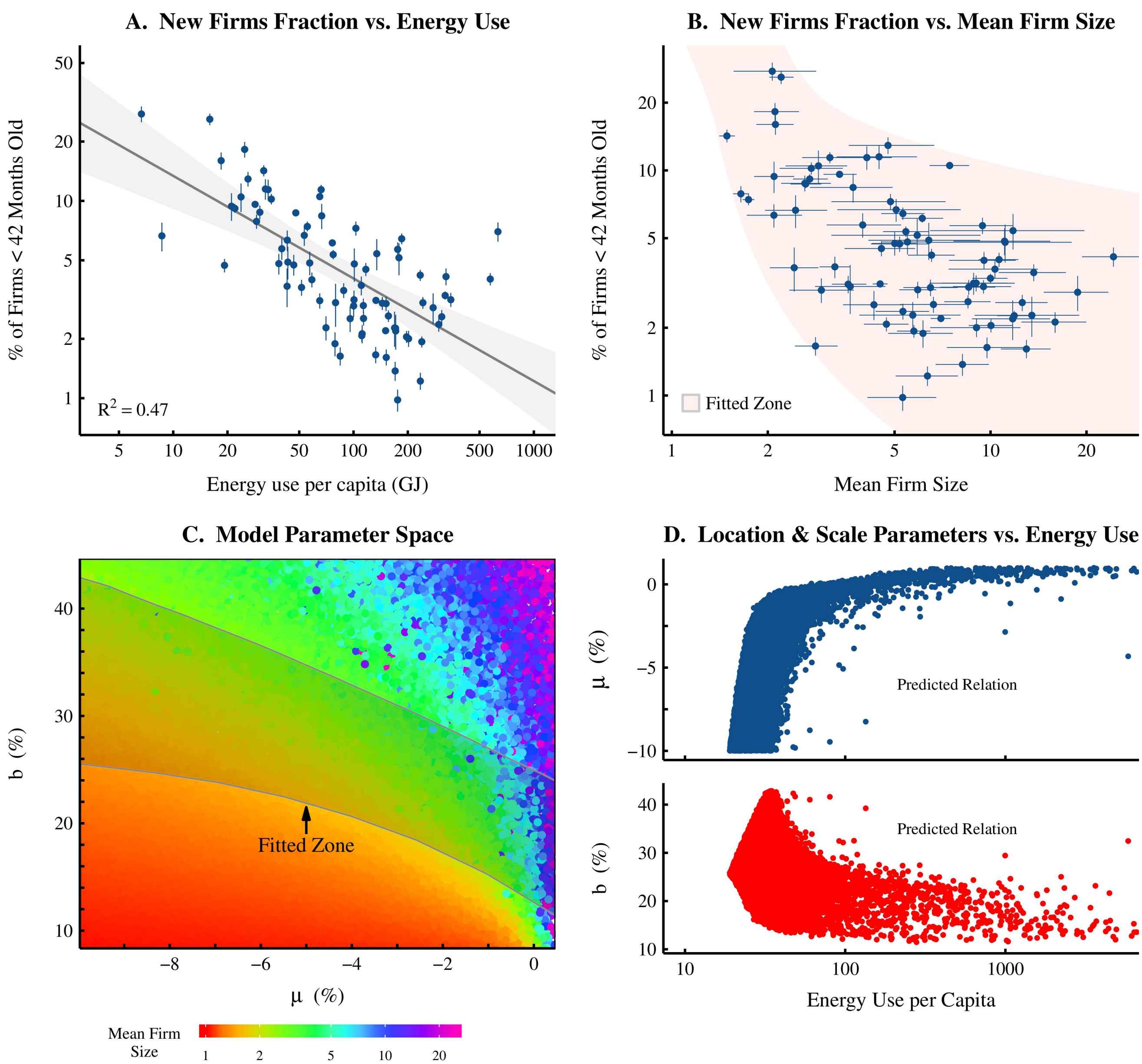

Fig 4. Using firm age data to estimate international firm dynamics. This figure demonstrates how firm age and mean size data can be used to restrict the parameter space of a stochastic model. This allows predictions to be made about the relation between energy use and firm dynamics. Panel A shows the country-level relation between the fraction of firms under 42 months old vs. energy use per capita (the grey region indicates the 99% confidence region of the regression). Panel B shows the country-level relation between the fraction of firms under 42 months old and mean firm size (error bars indicate 95% confidence intervals). The 'Fitted Zone' in Panel B shows the age-size relation produced by a stochastic model with a parameter range specifically chosen to capture the empirical data. Panel C shows the model's parameter space with the resulting mean firm size indicated by color. Using the regressed relation between mean firm size and energy use per capita (Fig 1C), modelled mean firm size is then transformed into an estimate for energy use per capita. The resulting relation between μ and b vs. energy use per capita (for data in the fitted zone only) is plotted in panel D.

doi:10.1371/journal.pone.0171823.g004

current age and size (i.e. a new, large firm likely grew rapidly, while an old, small firm likely grew slowly). Fig 4A shows how firm age is related to rates of energy consumption per capita. The dataset used here (the GEM database) does not report firm age directly. Instead, it reports whether or not a firm is under 42 months of age. I use this data in Fig 4A to calculate the fraction of firms that are under 42 months of age. This fraction tends to *decline* as energy use per capita increases.

This data clearly hints that a systemic relation exists between energy consumption and firm dynamics. In the following section, I use a stochastic model to make specific predictions about the form of this relation.

3.1 A stochastic model

The essence of all stochastic firm models is that growth is treated probabilistically. Each firm begins with some arbitrary initial size L_0. After every discrete time interval, the firm is subjected to a series of random 'shocks' (x_i) that perturb it from its initial size. In our model, these shocks are drawn randomly from a Laplace distribution. At any point in time, each firm's size $L(t)$ is equal to the initial size times the product of all shocks (Eq 1). If the time interval is years, then each shock can be interpreted as the annual growth rate (in fractional form).

$$L(t) = L_0 \cdot x_1 \cdot x_2 \cdot \ldots \cdot x_t \tag{1}$$

This basic Gibrat model is unstable unless additional stipulations are added (see S1 Appendix part E). I add a reflective lower bound that disallows firms from shrinking below the size $L = 1$ (this is sometimes called the Keston process [38–40]). As long as firm growth rates have a *downward* drift, the model will produce a stable firm size distribution. Using this model requires the following assumptions:

1. The firm size distribution is a power law.
2. Firm growth rates are independent of size.
3. New firms are all born at size $L = 1$.
4. The firm birth rate is equal to the firm death rate.
5. Firm growth rates come from a *Laplace* distribution.
6. The firm size distribution exists in an equilibrium.

Assumption 1 is necessary because the model produces a power law distribution (see S1 Appendix part F). Recent studies have found that firm size distributions in the United States [41] and other G7 countries [42] are approximately power laws. Less is known about developing countries. In S1 Appendix (part C), I demonstrate that the international data shown in Fig 1 is largely consistent with variations in a power law distribution, as are variations in the US firm size distribution over the last century.

Assumption 2 is a property of most stochastic firm growth models, and dates back to the work of Gibrat [9], who first found evidence that firm growth rates were independent of size. Since then, some studies have found that growth rate volatility tends to decline as firm size increases [10–12]). For the purposes of this model, I neglect this real-world complexity for the following reasons. First, firm growth rate studies use datasets (like Compustat) that are extremely biased towards large firms. Very little is known about the growth rates of small firms. In S1 Appendix (part D), I use the Compustat database (which is very biased towards large firms) to estimate how growth rates might vary with size in a non-biased sample. I find that declines in growth rate volatility are likely important for only a small minority of the largest firms. Furthermore, it is quite possible that the rate at which volatility declines with firm size varies by country and/or through time. However, good data (on which to base a model) is unavailable. Faced with this lack of knowledge, I choose to make the simplifying assumption that firm growth rates do not vary with size.

Assumptions 3 and 4 give meaning to the reflective lower bound. We can interpret this boundary as a firm birth/death zone. Any firm that passes below $L = 1$ is assumed to have 'died'. The reflection then represents the 'birth' of a new firm of size $L = 1$. Since all firms that 'die' are immediately 'reborn', this mechanism assumes that the firm birth rate equals the firm death rate. This interpretation of the model allows firm age to be defined as the period since the last reflection. In the real world, new firms are obviously not all born at size one; however, evidence suggests that they are much smaller than established firms [43, 44].

Regarding assumption 5, it is well established that the firm growth distribution has a tent-shape that can be modelled with the Laplace distribution [45, 46]. A Laplace (or double exponential distribution) has a sharper peak and fatter tails than a normal distribution. Various theories have been proposed to explain this phenomenon [47, 48]; however the causes of this growth rate distribution are exogenous to the current model.

Assumption 6 justifies testing the model against empirical data. Given some arbitrary initial conditions, the model will always approach a stable firm size distribution that is a function of only the growth rate distribution (provided that the stability conditions are met). Prior to arriving at equilibrium, there is *no relation* between the growth rate distribution and the firm size distribution (since any initial condition is possible). The equilibrium assumption justifies the link between growth rates and the firm size distribution.

3.2 Estimating variations in firm dynamics

The goal of this analysis is to estimate how firm dynamics (i.e. growth rate distributions) change with levels of energy consumption per capita. This estimation involves three steps. First, we must use appropriate empirical data to restrict the parameter space of the model. Second, we analyze how this parameter space relates to mean firm size. Finally, we extrapolate, from mean firm size, the relation between model parameters and energy use per capita.

Modelled growth rates are determined by the Laplace probability density function below, where μ and b are the location and scale parameters, respectively.

$$p(x) = \frac{1}{2b} e^{-|(x-\mu)/b|} \tag{2}$$

The parameter μ indicates the most probable growth rate, while b corresponds to growth rate volatility (larger b indicates greater volatility). Because μ and b are *free* parameters, we must use appropriate empirical data to restrict their range.

To do this, I use the empirical relation between the proportion of firms under 42 months of age and mean firm size (Fig 4B). A range of model parameters is chosen so that the resulting stochastic model produces the 'fitted zone' in Fig 4B. The corresponding parameter space of the model is shown in Fig 4C, with fitted zone parameters indicated by the shaded region. Equilibrium mean firm size for each μ and b coordinate is indicated by color.

The final step in the analysis is to use the regressed relation between mean firm size and energy use per capita (Fig 1C) to estimate energy consumption levels from modelled mean firm sizes (for data within the fitted zone only). We can then plot the resulting predicted relation between model parameters and energy use per capita (Fig 4D).

Our restricted stochastic model predicts the following: (1) μ should *increase* non-linearly with energy consumption; and (2) b should *decrease* non-linearly with energy consumption. In general terms, the model predicts that average firm growth rates should increase with energy consumption, while volatility should decline. This result represents a definitive prediction about how firm dynamics should vary with rates of energy consumption. Future empirical work can determine if this prediction is correct.

4 The 'why' question: Energy, technology and hierarchy

Any attempt to explain why institutions grow must first settle on the appropriate scale: do we attempt to explain why *individual* institutions grow, or do we concern ourselves only with changes in *average* size? The former is almost certainly a futile task, much like offering a general theory to explain why individual species go extinct. The answer is almost certainly, "It is complex". Species go extinct because of the complicated relation between their physiological characteristics and their environment. Likewise, individual institutions grow/shrink because of the complex relation between their characteristics and their environment (both biophysical and social).

The very success of stochastic firm growth models—in which *randomness* is the explanatory mechanism—suggests that the individual institution is not the appropriate domain for a 'why' explanation. Rather, we should be concerned with *groups* of institutions. This decision effectively bars the traditional toolbox of economic theory, which is to construct models based on simple postulates about the behavior of individual entities (consumers, firms, governments, etc.). Instead, we must rely on qualitative reasoning, tested against quantitative empirical evidence.

My explanation of the energy versus institution size relation builds on the 'social brain' hypothesis proposed by Dunbar [49]. According to this hypothesis, the size of the human brain inherently limits our ability to maintain social relations. As Turchin and Gavrilets note, social hierarchy offers a way around this limit [8]. Within a hierarchy, an individual must maintain relations with only his direct superior and direct subordinates. This means that a hierarchically organized group can grow in size *without* a corresponding increase in the number of required social relations. I argue that firms and governments are simply the modern embodiment of social hierarchy, and are used as tools of social coordination.

To connect social coordination to energy consumption, I explore the connection between energy use and technological scale. I argue that increases in energy consumption are associated with the use of increasingly large technologies. The construction, operation, and maintenance of these larger technologies, in turn, requires greater social coordination.

I formalize this reasoning in the joint hypotheses below. The order of these hypotheses is meant to show a line of reasoning, not necessarily a direction of causality.
Hypotheses

1. Increases in per capita energy consumption are accomplished (in part) through increases in technological *scale*.
2. Increases in technological scale require increases in *social coordination*.
3. Humans have a *limited* capacity to maintain social relations. Hence, egalitarian social coordination has strict limits.
4. *Social hierarchies* allow the scale of social coordination to grow without a corresponding increase in the number social relations.
5. Institutions (firms and governments) are dedicated social hierarchies.

In the following sections, I review the empirical evidence in support of each of these hypotheses.

4.1 Energy, technological scale and social coordination

My focus on technology (hypothesis A) is motivated both by theoretical arguments and by the empirical results in Fig 3.

From a theoretical (thermodynamic) perspective, energy 'consumption' is best thought of as a *conversion* process. For most organisms, this energy conversion process occurs *within* the body via cellular metabolism. Humans are unique among all other organisms in that we have developed many inorganic ways of harnessing energy *outside* our bodies. This inorganic energy consumption necessarily involves the use of man-made energy converters that transform primary energy into forms useful to humans. We call these man-made energy converters 'technology'. Since energy use is fundamentally related to technology, it makes sense to explore the ways in which technology relates to institution size.

On the empirical side, the fact that firm size scales with energy consumption both at the national and *sectoral* level (Fig 3) hints that technology mediates this relation. Unlike nation-states, which are defined by geographic boundaries, economic *sectors* are defined by a particular type of *activity*. Similar activities tend to use similar technologies. This is especially true as we move to the smallest manufacturing subsectors. With names like *Sawmills* (NAIC 321113), *Petroleum Refineries* (NAIC 32411), and *Iron Foundries* (NAIC 331511), these subsectors are practically *defined* by the technologies they use. This suggests that differences in energy use between such subsectors are related to differences in the technologies employed.

To illuminate the relation between energy and technology, consider the definitional statement that energy per capita (E_{pc}) is equal to total energy consumption (E) divided by population (P):

$$E_{pc} = \frac{E}{P} \tag{3}$$

Let us now define N as the total number of energy converters in society. By multiplying by N/N, we can rearrange Eq 3 to give:

$$E_{pc} = \frac{E}{N} \cdot \frac{N}{P} \tag{4}$$

Eq 4 indicates that energy use per capita is a function both of technological *scale* (E/N, average capacity per energy converter) and technological *density* (N/P, the number energy converters per capita).

In terms of social coordination, there is a fundamental difference between increasing energy consumption through technological density versus technological scale: the former is a *decentralized* process, while the latter requires *centralization*. Increasing energy use per capita through technological density involves independent changes in the behaviour of individuals, meaning it is an atomistic process. However, increasing energy consumption through technological scale requires the centralization of resources and human labor. Thus, it requires increases in social *coordination*.

As an example of a technological *density* process, consider the spread of household appliances (which are a type of end-use energy converter). The invention and widespread adoption of technologies such as the refrigerator, washer, dryer, microwave oven, and dishwasher vastly increased the number of energy converters per capita. At least on the consumer end (not the production end) this process was highly decentralized—individuals *independently* added more electronic devices to their lives.

As an example of a technological *scale* process, consider the changing scale of the industrial technologies shown in Table 1. Relative to their early prototypes, these technologies have undergone increases in scale by factors of one hundred (tanker ships) to factors of over a *million* (electric power plants). These changes in technological scale necessarily involve the increasing coordination of human labor. For instance, the largest oil refinery in the world, located in Jamnagar, India, employs 2500 people on site [50]. Rather than acting autonomously

Table 1. Scale increase of various industrial technologies.

Type	Early Prototype	Largest Today	Unit	Scaling Factor
Electric Power Plant	0.0125	2 2500	megawatts	1.80×10^6
Oil Refinery	5.5	1 240 000	barrels per day	2.24×10^5
Aluminium Smelter	5.7	1 060 000	tonnes per year	1.86×10^5
Internal Combustion Engine	0.75	107 390	horsepower	1.43×10^5
Mining Excavator	380	2 324 0000	cubic meters per day	6.12×10^4
Blast Furnace	0.3	5 500	cubic meters	1.83×10^4
Tanker Ship	1809	260 859	gross tonnage	1.44×10^2

This table shows the size of 7 selected industrial technologies at their earliest stage of development ('Early Prototype') and at the largest scale existing today. Column 5 shows the scaling factor between the largest and early technologies (largest/early). Technologies are ranked in descending order of scaling factor. For data sources, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.t001

(like the users of consumer electronics), these individuals must coordinate their actions over a wide range of different tasks. This suggests that increases in technological scale require an increase in social coordination.

But to what degree are increases in energy use per capita actually achieved through increases in technological scale? Given the complexity of technological change, this question is difficult to answer at a general level (for all technologies). Instead of a general test of hypothesis A, I present here a case study of electricity production and consumption in the United States (Fig 5A and 5B). The results of this case study indicate that increases in technological scale have played an important role in meeting increases in per capita electricity use over the last century.

Fig 5A shows how the indexed change in US electricity use per capita relates to the indexed change in mean power plant size (as measured by nameplate capacity). Over the last 100 years, the two series tracked together quite closely, with both electricity use and power plant size increasing rapidly between 1920 and 1980 and plateauing thereafter. How important was this change in technological scale for meeting per capita demand? To answer this question, Fig 5B plots the indexed ratio of mean power plant size to electricity use per capita. This ratio indicates the fraction of electricity use per capita growth that was met by increases in power plant capacity. Between 1920 and 2015, increases in power plant capacity accounted for roughly *half* of the total increase in electricity use per capita.

In the US electricity generation sector, increases in technological scale obviously played a major role in meeting increases in per capita electricity consumption. Was this increase in scale accompanied by a corresponding increases in the scale of social coordination (hypothesis B)? Answering this questions requires that we first define what we mean by the 'scale' of social coordination, and specify how this relates to a given technology.

I define the 'scale' of social coordination as the number of people required to construct, maintain, and operate a specific technology. For measurement purposes, however, I limit my analysis only to *construction* labor time. This decision is driven primarily by data availability (and lack thereof). For the most part, published power plant data focuses almost exclusively on *costs*, and primarily on the cost of *construction*. Fortunately, with a few simplifying assumptions, construction cost data can be used to estimate construction labor time. I use this latter metric to quantify the scale of social coordination associated with a given power plant.

To estimate construction labor time from costs, I first note that by the rules of double-entry accounting, all costs eventually become someone's *income*. If we assume that all income accrues to labor (i.e. we neglect capitalist income) then we can divide the total cost of a project

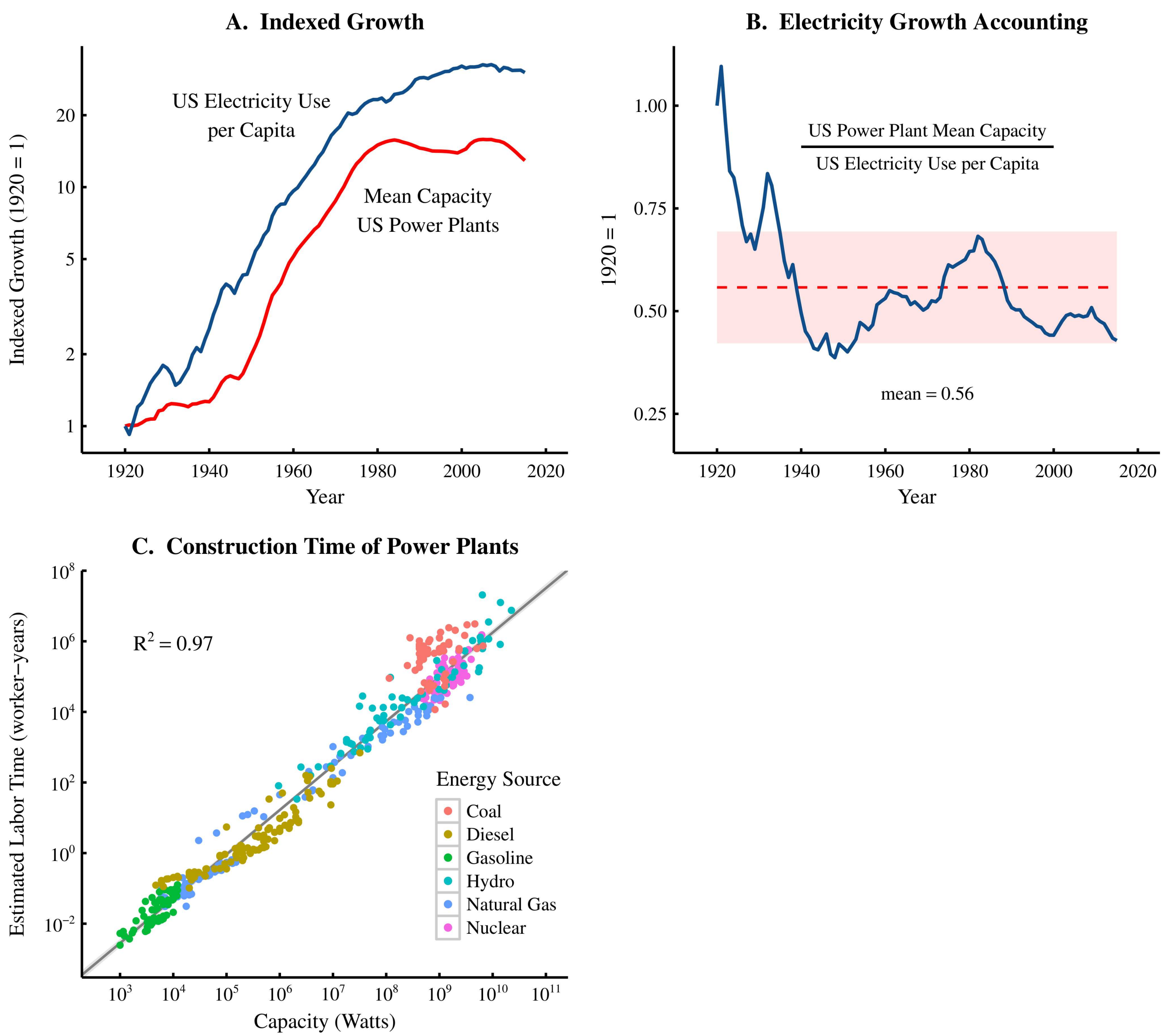

Fig 5. Technological scale and social coordination in electricity generation. Panel A shows the time-series relation between the mean capacity of US power plants and US electricity use per capita. Both series are indexed to 1 in the year 1920 in order to show relative growth. Power plants tend to get larger as electricity use per capita increases increases. Panel B shows the fraction of US per capita electricity use growth (since 1920) that was met by increases in mean plant size. The dashed line indicates the mean over the period 1920–2015, while the shaded region shows the standard deviation. Panel C shows the relation between power plant capacity and the estimated construction labor time. The entire range of electricity generation technology is included in this plot—from the smallest gasoline generators to the largest hydroelectric power plants. Different primary energy sources are indicated by color. Data is modelled with a power law. Grey regions indicate the 99% confidence region of the regression. For sources and methodology, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.g005

by an estimate of the average wage to obtain a rough estimate of the total labor time involved. I use GDP per capita as a measure of average income, giving Eq 5 as my method for estimating labor time.

$$\text{Labor Time} \approx \frac{\text{Total Cost}}{\text{GDP per capita}} \tag{5}$$

Although this method contains some implicit bias/error, I show in S1 Appendix (part G) that it is unlikely that this bias/error affects the integrity of the results (largely due to the vast size range of power plant studied here).

Fig 5C applies this method to estimate the construction labor time of approximately 500 different power plants and generators. The capacity of these plants/generators ranges over 7 orders of magnitude—from the smallest gas-powered generator (1000 watts) to the largest hydroelectric dams (the 22.5 gigawatt Three Gorges Dam). Different energy sources are indicated by color. The results show a strong scaling relation between plant capacity and construction labor time. This indicates that the scale of social coordination necessary to build a power plant is strongly related to the plant's energy conversion capacity.

To summarize, our case study of the electricity generation sector is consistent with both hypothesis A and B. We find that increases in power plant scale have played an important role in meeting increases in US per capita electricity consumption (hypothesis A). Furthermore, we find that power plant size is strongly related to construction labor time—our measure of the scale of social coordination (hypothesis B).

Admittedly, a case study of a single technology represents limited evidence. However, the vast scaling of the other technologies shown in Table 1 indicates that this line of reasoning has promise. To continue my arguments, I will assume that the findings of this case study can be generalized to many other technologies. The result (we assume) is a that increases in energy consumption require a generalized increase in the scale of human social coordination. The question, then, is how is this coordination accomplished?

4.2 Social coordination and human biology

Social coordination can conceivably be achieved in many different ways (customs, markets, institutions, etc.). Thus, an increase in social coordination does not necessarily imply an increase in firm and government size. Why, then, have these institutions increased in size as energy consumption increases? Hypotheses C-E propose a chain of reasoning explaining why institutions are the most effective way of organizing large groups of people. The key to this reasoning is hypothesis C: humans have a *limited* ability to maintain social relations.

The evidence for this hypothesis comes primarily from the work of anthropologist Robin Dunbar, who has uncovered a startling relation between primate brain size and mean group size [7]: primate species with larger brains (as measured by the relative size of the neocortex) tend to live in larger groups. Dunbar has developed this finding into what he calls the *social brain hypothesis*: "primates evolved large brains to manage their unusually complex social system[s]" [49].

The implication of Dunbar's findings is that the size of the human brain places limitations on the number of social relations that an individual is able to maintain. Dunbar uses his primate data to predict a mean human group size of about 150. While this number should be considered exploratory, Dunbar notes that early egalitarian societies had group sizes around this order of magnitude [51].

A key feature of egalitarian organization is that any member of a group may maintain relations with any other member of the group. Thus, the number of possible social relations increases linearly with group size. Given the hypothesized limitations in the human ability to maintain social relations, it follows that egalitarian social organization is not an effective method for coordinating large numbers of people.

One way of increasing group size beyond Dunbar's number is to organize groups in a way that *limits* human interaction. Turchin and Gavrilets note that this is a key feature of social hierarchies, which are characterized by a treelike chain of command [8]. Within a hierarchy an individual must maintain social relations only with his direct superior and direct inferiors. Thus, hierarchy allows group size to grow without any corresponding increase in the number of human relations (hypothesis D).

As evidence for this line of reasoning, Turchin and Gavrilets demonstrate that a strong correlation exists between the population of historical agrarian empires and the number of administrative (hierarchical) levels within their respective governments. Similarly, Hamilton et al. find a strong relation between population size and the number of hierarchical levels with various hunter-gatherer societies [52]. This evidence suggests that social hierarchy is a common tool used for increasing the scale of social coordination.

4.3 Hierarchy and institution size

Social hierarchies have taken many different forms at different points in human history. For instance, in many pre-state societies, social hierarchy took the form of the chiefdom. In middle-ages Europe, the feudal manor was the principle unit of hierarchy. In the modern era, I argue that business firms and governments are the principle unit of social hierarchy (hypothesis E). To test this hypothesis, I focus only on firms.

The implication of hypothesis E is that increasing firm size constitutes an *investment in social hierarchy*. If this reasoning is correct, then mean firm size should be an indicator of the relative 'top heaviness' of a society. Why? Hierarchies tend to become more top heavy as they become larger—the fraction of individuals in the upper echelons tends to grow as the size of the hierarchy increases. Thus, if firms are the modern embodiment of social hierarchy, then mean firm size should be related to the relative size of the upper social echelon.

Since the upper echelons of a hierarchy are almost exclusively involved in managing the activities of other people, it seems sensible to use the *management* profession as a metric for the size of this top cohort. Thus, if hypothesis E is correct, we expect that increases in mean firm size should be associated with an increase in the employment share of managers.

To refine this prediction, I develop a hierarchical firm model of society (Fig 6) based on the following assumptions:

1. All firms are 'ideal' hierarchies with a single span of control.
2. All individuals in and above the third hierarchical level are considered 'managers'.
3. The firm size distribution is a power law.

Why assume that management begins at the third hierarchical level? Obviously, individuals within the lowest hierarchical level have no management responsibilities. Those in the second hierarchical level can be thought of as 'working supervisors'—individuals who have some supervisory responsibilities but who spend a majority of their time engaged in 'production' [53]. I assume that individuals in and above the third hierarchical level are devoted mostly to managing the work of others.

This model predicts that the management fraction of employment should grow non-linearly with firm size, eventually approaching an asymptote defined only by the span of control. If the span of control is s, then the asymptote occurs at $1/s^2$ (see S1 Appendix (part H) for the details of this calculation).

In Fig 7 I test this model at the international level. Fig 7A and 7B plot the country-level relation between the management fraction of employment versus mean firm size (the two plots show different occupation classification regimes). Empirical data is shown in black, while model predictions are shown in the background with the span of control indicated by color.

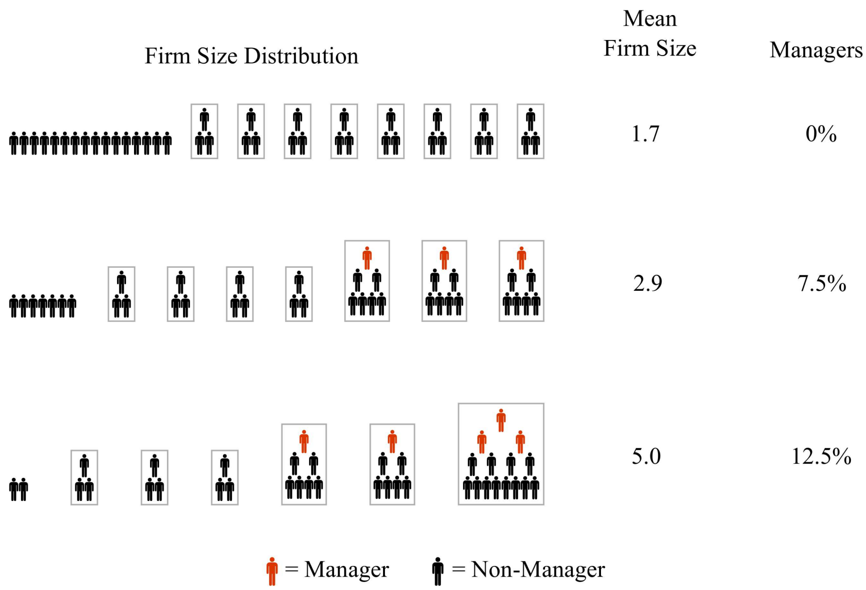

Fig 6. The growth of management as a function of the firm size distribution. This figure graphically demonstrates how the management fraction increases with firm size (assuming firms are 'ideal hierarchies'). Firms are indicated by boxes (with the exception of single-person firms) with a worker's hierarchical position shown vertically. The span of control—defined as the size ratio between adjacent hierarchical levels—is constant for all firms. In this picture, the span of control is 2. Managers (red) are assumed to be all individuals in and above the third hierarchical level. To maintain simplicity, this graphic does not use a power law firm size distribution.

doi:10.1371/journal.pone.0171823.g006

Different mean firm sizes are produced by varying the exponent of the firm size power law distribution (for a technical discussion of this model, see S1 Appendix part H).

The model nicely reproduces the observed relation between mean firm size and the management fraction of employment. However, this fit is achieved by freely manipulating the span of control parameter. Thus, it is important to check that the modelled span of control range is consistent with the span range for *real* firms.

Ideally we would be able compare the span range of the model to the span distribution of a large, global sample of firms. Unfortunately, data constraints make this impossible. Due to the proprietary nature of firm personnel data, only a handful of studies have analyzed firm hierarchies. Fig 7C shows data from 12 such studies that together sample firms from 7 different nations (Denmark, Japan, Netherlands, Portugal, the United Kingdom, the United States, and Sweden). The resulting firm sample gives relatively good coverage of wealthy nations, but unfortunately does not include any firms from developing countries (due to the lack of available studies). For a summary of the data sources, see S1 Appendix (part A).

Boxplots in Fig 7C correspond to the span of control range found by each study. Note that the data is a mixture of case studies of single firms and aggregate studies that analyze the structure of many different firms. While these aggregate studies give better scope than the case studies, many focus only on the upper levels of the hierarchy (where data is more easily obtained). The important finding in Fig 7C is that the model's fitted span of control range is consistent with the available empirical data.

To summarize these findings, a simple hierarchical firm model of society is able to replicate the observed relation between mean firm size and the management share of employment. The changes in mean firm size are achieved by varying the exponent of a firm size power law distribution, while the management fraction of employment is fitted by 'tuning' the span of control

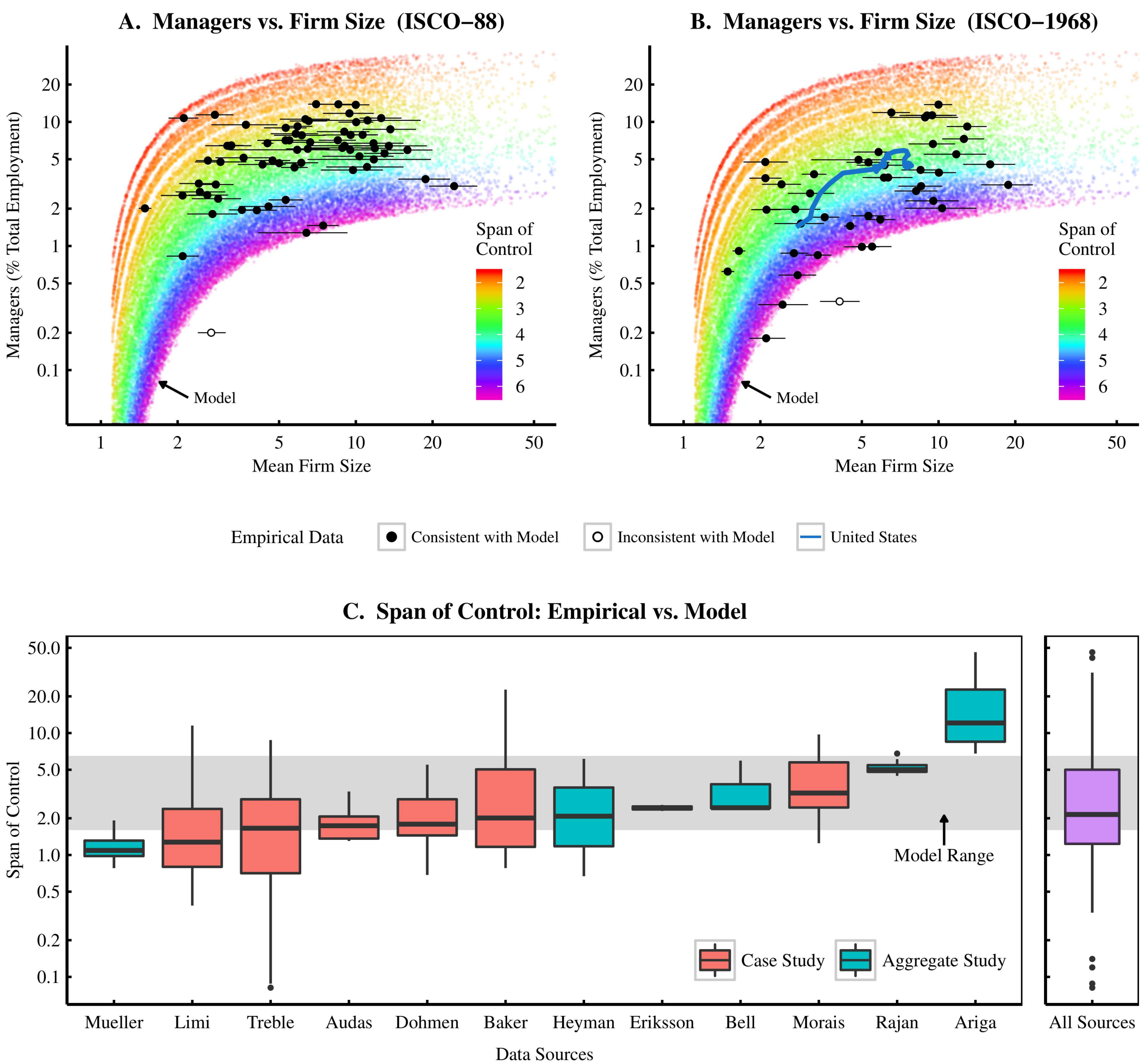

Fig 7. Testing the hierarchical model of the firm using managment share of total employment. Panels A and B plot the country-level relation between the management fraction and mean firm size. Modelled data is also shown in the background, with the span of control indicated by color. Panels A and B use different (incommensurable) classification methodologies for 'management'. Panel A uses ISCO-88 (which includes legislators, senior officials and managers) while panel B uses ISCO-1968 (which includes administrative and managerial workers). Error bars indicate the 95% confidence intervals for mean firm size. Panel C compares the span of control range from the model to the span distribution found by 12 different empirical studies. Red boxplots indicate case studies, and show the span of control distribution within a *single firm*. Blue boxplots indicate aggregate studies and show the span of control distribution across many different firms. The span of control distribution across all 12 studies is shown on the right. For sources and methodology, see S1 Appendix (part A).

doi:10.1371/journal.pone.0171823.g007

range (assumed to be the same both within and between all modelled firms). Importantly, the resulting fitted span range is consistent with the existing empirical data on the internal structure of the firm. The success of this model gives support to hypothesis E, and suggests that increases in mean firm size are characteristic of a generalized increase in social hierarchy.

4.4 Causality

I have proposed hypotheses A-E as a chain of reasoning connecting energy consumption to institution size. But which way does causation run? Do increases in energy consumption *cause* institutions to become larger, or is the reverse true? As I discuss below, it seems likely that causation runs in *both* directions.

Although hypotheses A-E are framed in terms of *increases* in energy use (and institution size), I think that a discussion of causation is clearer when framed in terms of *constraints* and *decline*. For instance, I think it *must* be the case that energy constraints place limits on institution size. This is for the simple reason that energy conversion technology is *useless* without an energy input. I have proposed that large institutions provide the social coordination necessary to build and operate large technologies. But without sufficient energy input, these technologies cannot be operated, and the institution's raison d'être ceases to exist. Imagine how long a large steel firm would stay in business if there was not enough coke to fuel its large blast furnaces. This line of thinking implies that a decline in energy consumption (due to scarcity) can *cause* a decline in institution size.

However, recent history (the collapse of the Soviet Union) suggests that causality can operate in the *reverse* direction. Fig 8 shows energy and government employment share trends in six nation-states that emerged after the dissolution of the USSR. In the aftermath of the Soviet collapse, these six countries experienced *drastic* reductions in both government size and

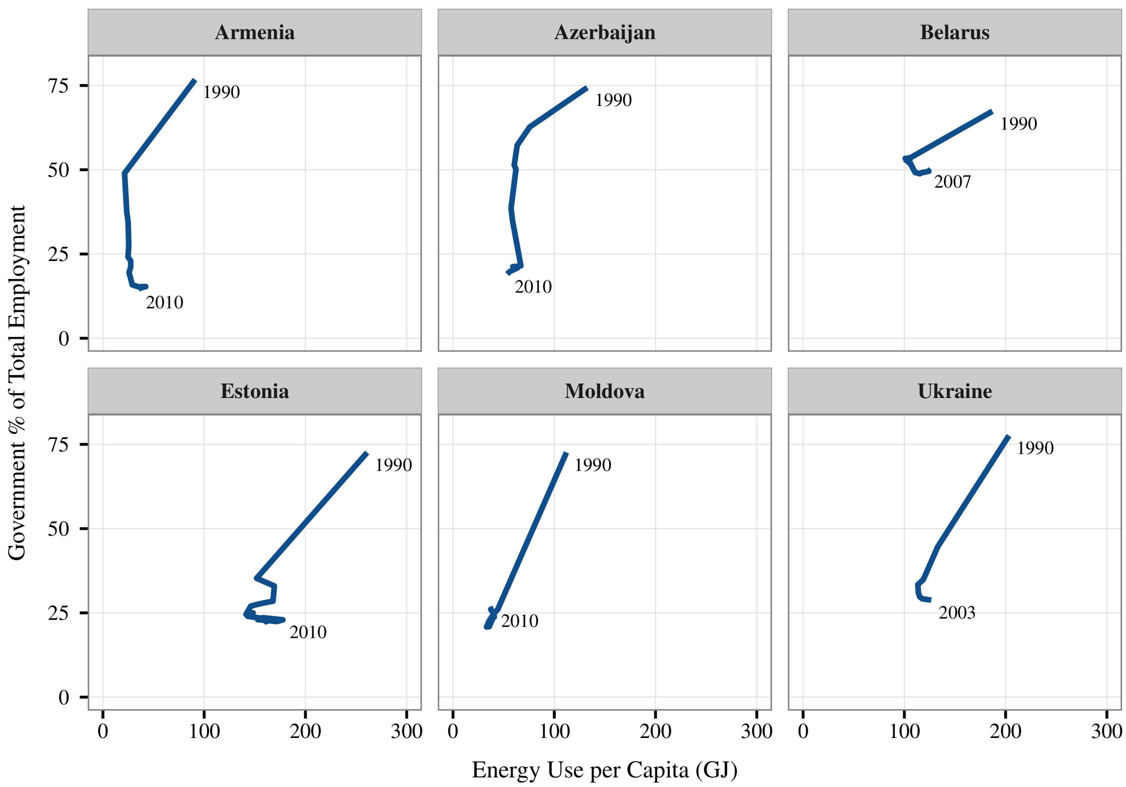

Fig 8. A case study in causality: The collapse of the soviet union. This figure tracks the path through time of six nations that emerged after the collapse of the Soviet Union (in 1990–91). As the collapse unfolded, the fraction of people employed by the government shrank rapidly, as did energy use per capita. Since the USSR collapse was an institutional crisis (not an energy crisis), this suggests that at least in this case, causality runs from institution size to energy consumption.

doi:10.1371/journal.pone.0171823.g008

energy use. During this period, there was no global energy shortage, meaning biophysical energy constraints can likely be ruled out as a causal factor. Instead, it seems likely that institutional collapse is the driving factor here.

This case is illustrative because the Soviet economy relied on an unusually high degree of government control of production, placing an enormous amount of power in the hands of a single institution. Not surprisingly, the collapse of this institution led to social chaos and widespread economic decline. I think this shows quite clearly that institutional collapse can cause a decline in energy consumption.

The argument that causation can operate in *both* directions suggests that energy use and institution size exhibit a *feedback* relation (rather than linear causality). One possible avenue for furthering this research is to use systems modelling. Ugo Bardi has shown that a simple adaptation of the Lotka–Volterra equations can be used to model the relation between energy extraction and a technological stock [54]. A plausible line of future research would be to add institution size to this type of model.

It is also important to note that changes in energy use and institution size occur alongside other social changes, the two most obvious being urbanization and changes in sector composition [35]. It seems likely that these phenomena are all interrelated—part of a complex process of social change accompanying changes in energy consumption. In S1 Appendix (part I), I use an adaptation of the hierarchical firm model (used in Fig 7) to explore the institution size constraints that are inherent in the sectoral composition of agrarian societies. The results offer a promising way of broadening our understanding of why energy use is related to institution size.

5 Conclusions

All life on earth is united by a common struggle—a "struggle for free energy available for work" [55]. The ability to harness energy places key constraints on the structure of life, from the level of the cell [56], to the organism [57, 58], to the ecosystem [59]. Within this unifying context, it seems plausible that the structure of human society ought to be related to the ability to harness energy.

Based on this line of reasoning, a branch of scholarship has emerged that studies the role of energy in human societies [36, 60–65]. However, to my knowledge, this paper is the first to explicitly connect energy use with institution size. This connection is important because it is not easily explained by existing institution size theories, which focus mostly on the monetary incentives for institution growth.

I have offered a new theory of institution size that is rooted in human biology, and the theorized limitations of our ability to maintain social relations. I have proposed that institutions (firms and governments) are social hierarchies that serve to increase the scale of social coordination beyond that which is possible through egalitarian relations. I have argued that increases in energy consumption require a general increase in the scale of social coordination, and that increases in technological scale are a plausible reason for this connection. There is, of course, no need for increases in technological scale to be the *only* reason why social coordination increases with energy use—it is simply the easiest to study.

An important prediction of this theory is that increases in energy consumption are associated with a general increase in social hierarchy, meaning power is concentrated in the hands of fewer and fewer people. Although this starkly contradicts neoclassical economic theory, it is consistent with the power-based approach to political economy offered by Nitzan and Bichler [17]. If concentrations of power are at the heart of increases in energy consumption, then the theory developed here may be useful for studying a broad range of modern political economic phenomena.

Supporting information

S1 Appendix. Energy and institution size appendices. Contains information on data sources and methods. It also contains extra analysis referenced in the main paper.
(PDF)

S1 File. Data and code. This zip file contains raw and final data for all analysis conducted in this paper. It also includes R code used for modelling.
(ZIP)

Acknowledgments

The method used in this paper owes a great deal to the work of Jonathan Nitzan, who encouraged me to abandon the use of real GDP, and who made me aware of many of the databases used in this paper. I would also like to thank Ellie Perkins and Mark Thomas, as well as the two anonymous reviewers, for their helpful comments on drafts of this paper. I would also like to thank Garry and Grace Fix for their proofreading skills.

Author contributions

Conceptualization: BF.

Formal analysis: BF.

Investigation: BF.

Methodology: BF.

Software: BF.

Validation: BF.

Visualization: BF.

Writing – original draft: BF.

Writing – review & editing: BF.

References

1. Cottrell F. Energy & Society (Revised): The Relation Between Energy, Social Change, and Economic Development. Bloomington: AuthorHouse; 2009.
2. Hall C, Tharakan P, Hallock J, Cleveland C, Jefferson M. Hydrocarbons and the evolution of human culture. Nature. 2003; 426(6964):318–322. doi: 10.1038/nature02130 PMID: 14628060
3. Soddy F. Virtual Wealth and Debt: the Solution of the Economic Paradox. London: George Allen & Unwin; 1926.
4. White LA. Energy and the evolution of culture. American Anthropologist. 1943; 45(3):335–356. doi: 10.1525/aa.1943.45.3.02a00010
5. Kondepudi DK, Prigogine I. Modern Thermodynamics: from Heat Engines to Dissipative Structures. Chichester: John Wiley & Sons; 1998.
6. Adams RN. Man, Energy, and Anthropology: I Can Feel the Heat, But Where's the Light? American Anthropologist. 1978; 80(2):297–309.
7. Dunbar RI. Neocortex size as a constraint on group size in primates. Journal of Human Evolution. 1992; 22(6):469–493. doi: 10.1016/0047-2484(92)90081-J
8. Turchin P, Gavrilets S. Evolution of complex hierarchical societies. Social Evolution and History. 2009; 8(2):167–198.
9. Gibrat R. Les inegalites economiques. Recueil Sirey; 1931.

10. Hart PE. The size and growth of firms. Economica. 1962; 29(113):29–39. doi: 10.2307/2601518
11. Hymer S, Pashigian P. Firm size and rate of growth. The Journal of Political Economy. 1962; 70 (6):556–569. doi: 10.1086/258716
12. Singh A, Whittington G. The size and growth of firms. The Review of Economic Studies. 1975; 42 (1):15–26. doi: 10.2307/2296816
13. De Wit G. Firm size distributions: An overview of steady-state distributions resulting from firm dynamics models. International Journal of Industrial Organization. 2005; 23(5):423–450. doi: 10.1016/j.ijindorg.2005.01.012
14. Sutton J. Gibrat's legacy. Journal of Economic Literature. 1997; 35(1):40–59.
15. Coase RH. The nature of the firm. Economica. 1937; 4(16):386–405. doi: 10.1111/j.1468-0335.1937.tb00002.x
16. Geroski PA. The growth of firms in theory and in practice. Center for Economic Policy Research. 1999;2092.
17. Nitzan J, Bichler S. Capital as Power: A Study of Order and Creorder. New York: Routledge; 2009.
18. Lucas RE Jr. On the size distribution of business firms. The Bell Journal of Economics. 1978; 9(2):508–523. doi: 10.2307/3003596
19. Gomes PM, Kuehn Z. Human capital and the size distribution of firms. IZA Discussion Papers Series. 2014; (8268).
20. Poschke M. The firm size distribution across countries and skill-biased change in entrepreneurial technology. IZA Discussion Papers Series. 2014;7991.
21. Barney J. Firm resources and sustained competitive advantage. Journal of management. 1991; 17 (1):99–120. doi: 10.1177/014920639101700108
22. Peteraf MA. The cornerstones of competitive advantage: A resource-based view. Strategic Management Journal. 1993; 14(3):179–191. doi: 10.1002/smj.4250140303
23. Priem RL, Butler JE. Tautology in the resource-based view and the implications of externally determined resource value: further comments. Academy of Management Review. 2001; 26(1):57–66. doi: 10.2307/259394
24. Meltzer AH, Richard SF. A rational theory of the size of government. The Journal of Political Economy. 1981; 89(5):914–927. doi: 10.1086/261013
25. Peltzman S. The growth of government. Journal of Law and Economics. 1980; 23(2):209–287. doi: 10.1086/466962
26. Brennan G, Buchanan JM. The power to tax: Analytic foundations of a fiscal constitution. New York: Cambridge University Press; 1980.
27. Niskanen WA. Bureaucracy and Representative Government. Piscataway, NJ: Transaction Publishers; 1974.
28. Henrich J, Boyd R, Bowles S, Camerer C, Fehr E, Gintis H, et al. In search of homo economicus: behavioral experiments in 15 small-scale societies. American Economic Review. 2001; 91(2):73–78. doi: 10.1257/aer.91.2.73
29. Beck T, Demirgus-Kunt A, Maksimovic V. Financial and legal institutions and firm size. The World Bank Development Research Group. 2003;2997.
30. Biggs T, Oppenheim J. What drives the size distribution of firms in developing countries. Employment and Enterprise Policy Analysis, Discussion paper. 1986;(6).
31. Gollin D. Nobody's business but my own: Self-employment and small enterprise in economic development. Journal of Monetary Economics. 2008; 55(2):219–233. doi: 10.1016/j.jmoneco.2007.11.003
32. Georgescu-Roegen N. The Entropy Law and the Economic Process. Cambridge, Mass.: Harvard University Press; 1971.
33. Ayres R, Warr B. Exergy, power and work in the US economy, 1900–1998. Energy. 2003; 28(3):219–273. doi: 10.1016/S0360-5442(02)00089-0
34. Cleveland C, Costanza R, Hall C, Kaufmann R. Energy and the US economy: a biophysical perspective. Science. 1984; 225(4665):890–897. doi: 10.1126/science.225.4665.890 PMID: 17779848
35. Fix B. Rethinking Economic Growth Theory from a Biophysical Perspective. Hall C, editor. New York: Springer; 2015.
36. Hall C, Klitgaard K. Energy and the Wealth of Nations: Understanding the Biophysical Economy. New York: Springer; 2012.
37. Simon HA, Bonini CP. The size distribution of business firms. The American Economic Review. 1958; 48(4):607–617.

38. Biham O, Malcai O, Levy M, Solomon S. Generic emergence of power law distributions and Levy-Stable intermittent fluctuations in discrete logistic systems. Physical Review E. 1998; 58(2):1352. doi: 10.1103/PhysRevE.58.1352

39. Gabaix X. Zipf's Law for Cities: An Explanation. The Quarterly Journal of Economics. 1999; 114 (3):739–767. doi: 10.1162/003355399556133

40. Kesten H. Random difference equations and Renewal theory for products of random matrices. Acta Mathematica. 1973; 131(1):207–248. doi: 10.1007/BF02392040

41. Axtell RL. Zipf distribution of US firm sizes. Science. 2001; 293:1818–1820. doi: 10.1126/science.1062081 PMID: 11546870

42. Gaffeo E, Gallegati M, Palestrini A. On the size distribution of firms: additional evidence from the G7 countries. Physica A: Statistical Mechanics and its Applications. 2003; 324(1–2):117–123. doi: 10.1016/S0378-4371(02)01890-3

43. Angelini P, Generale A. On the evolution of firm size distributions. The American Economic Review. 2008; 98(1):426–438. doi: 10.1257/aer.98.1.426

44. Cabral LM, Mata J. On the evolution of the firm size distribution: Facts and theory. American economic review. 2003; 93(4):1075–1090. doi: 10.1257/000282803769206205

45. Bottazzi G, Secchi A. Explaining the distribution of firm growth rates. The RAND Journal of Economics. 2006; 37(2):235–256. doi: 10.1111/j.1756-2171.2006.tb00014.x

46. Stanley MH, Amaral LA, Buldyrev SV, Havlin S, Leschhorn H, Maass P, et al. Scaling behaviour in the growth of companies. Nature. 1996; 379(6568):804–806. doi: 10.1038/379804a0

47. Bottazzi G, Secchi A. Why are distributions of firm growth rates tent-shaped? Economics Letters. 2003; 80(3):415–420.

48. Fu D, Pammolli F, Buldyrev SV, Riccaboni M, Matia K, Yamasaki K, et al. The growth of business firms: Theoretical framework and empirical evidence. Proceedings of the National Academy of Sciences of the United States of America. 2005; 102(52):18801–18806. doi: 10.1073/pnas.0509543102 PMID: 16365284

49. Dunbar RI. The social brain hypothesis and its implications for social evolution. Annals of human biology. 2009; 36(5):562–572. doi: 10.1080/03014460902960289 PMID: 19575315

50. About Jamnagar; 2011. Available from: http://web.archive.org/web/20111027062343/http://www.ril.com/downloads/pdf/about_jamnagar.pdf.

51. Dunbar RI. Coevolution of neocortical size, group size and language in humans. Behavioral and Brain Sciences. 1993; 16(04):681–694. doi: 10.1017/S0140525X00032325

52. Hamilton MJ, Milne BT, Walker RS, Burger O, Brown JH. The complex structure of hunter–gatherer social networks. Proceedings of the Royal Society of London B: Biological Sciences. 2007; 274 (1622):2195–2203. doi: 10.1098/rspb.2007.0564

53. Strauss G. The Changing Role of the Working Supervisor. The Journal of Business. 1957; 30(3):202–211. doi: 10.1086/294147

54. Bardi U, Lavacchi A. A simple interpretation of Hubbert's model of resource exploitation. Energies. 2009; 2(3):646–661. doi: 10.3390/en20300646

55. Boltzmann L. Populare Schriften. SEVERUS Verlag; 2011.

56. Lane N, Martin W. The energetics of genome complexity. Nature. 2010; 467(7318):929–934. doi: 10.1038/nature09486 PMID: 20962839

57. Ahlborn BK. Thermodynamic Limits of Body Dimension of Warm Blooded Animals. Journal of Non-Equilibrium Thermodynamics. 2005; 25(1):87–102.

58. McNab BK. Resources and energetics determined dinosaur maximal size. Proceedings of the National Academy of Sciences. 2009; 106(29):12184–12188. doi: 10.1073/pnas.0904000106

59. Odum HT. Systems Ecology; an introduction. New York: Wiley; 1983.

60. Ayres R, Warr B. The Economic Growth Engine: How Energy and Work Drive Material Prosperity. Northampton, MA: Edward Elgar Pub; 2009.

61. Chaisson E. Cosmic Evolution: The Rise of Complexity in Nature. Cambridge, Mass.: Harvard University Press; 2002.

62. Giampietro M, Mayumi K, Sorman A. The Metabolic Pattern of Societies: Where Economists Fall Short. New York: Routledge; 2012.

63. Kummel R, Lindenberger D, Eichhorn W. The productive power of energy and economic evolution. Indian Journal of Applied Economics. 2000; 8(2):1–26.

64. Pimentel D, Pimentel MH. Food, Energy, and Society, Third Edition. New York: CRC Press; 2007.

65. Smil V. Energy in nature and society: general energetics of complex systems. Cambridge, MA: MIT press; 2008.

Appendices for Energy and Institution Size

Contents

Acronyms

BEA US Bureau of Economic Analysis

BLS US Bureau of Labor Statistics

EIA US Energy Information Agency

HSUS Historical Statistics of the United States

ILO International Labour Organization

GEM Global Entrepreneurship Monitor

WBES World Bank Enterprise Survey

A Sources and Methodology

Electricity Use per Capita

US electricity use is from HSUS table Db228 (1920 - 1948) spliced to EIA table 7.1, *Electricity End Use, Total* (1949-2015). US population is from Maddison [1] (1920-2009) and World Bank series SP.POP.TOTL (2010-2015).

Energy Use per Capita – International

International energy use per capita data is from the World Bank (series EG.USE. PCAP.KG.OE).

Energy Use per Capita – United States

US total energy consumption is from HSUS, Tables Db164-171 (1890-1948) and EIA Table 1.3 (1949-2012). US population is from Maddison [1] (1890-2009) and World Bank series SP.POP.TOTL (2010-2012).

Energy Use per Capita – US Industry

US Industry energy use is from EIA Table 2.1 (Energy Consumption by Sector). Industry employment is from BEA Table 6.8B-D (Persons Engaged in Production by Industry), where 'Industry' is defined to include Mining, Manufacturing and Construction.

Energy Use per Capita – US Manufacturing Subsectors

US manufacturing sub-sector energy use is from EIA Manufacturing Energy Consumption Survey Table 1.1 (First Use of Energy for All Purposes) 2002, 2006, and 2010. Manufacturing subsector employment is from Statistics of U.S. Businesses (US 6 digit NAICS) for 2002, 2006, and 2010.

Firm Age Composition

The fraction of firms under 42 months old (3.5 years) is calculated from the GEM dataset aggregated over the years 2001-2011 (data series *babybuso*). This series gives true/false values for whether or not a given firm is under 42 months old. Uncertainty in this data is estimated using the bootstrap method [2].

Firm Age Model

In order to model firm age accurately, I use a time step interval of 0.5 years (this allows us to calculate firms under 3.5 years so that we can compare to GEM data). However, most empirical data on firm growth rates are reported with a time interval of 1 year. In order to facilitate comparison with empirical data, I convert model growth rate parameters (μ and σ) into the equivalent parameters for a time step of 1 year. Code for this conversion process is provided in the supplementary material.

Firm size – International

International mean firm size data is estimated using the Global Entrepreneurship Monitor (GEM) database, series *omnowjob*. Data is aggregated over the years 2000-2011. In order to account for the over-representation of large firms, *I remove firms with more than 1000 employees from the database* (see Appendix B for a discussion).

This 'truncation' amounts to removing the top 0.2% of firms in the GEM database. The effects of this truncation on GEM country samples are shown in Figure 1. For 35 out of 89 counties, this has *no effect*, since these country samples do not contain firms larger than 1000 employees. The median percentage of firms removed (by country sample) is 0.01%. For a small number of countries, this truncation removes more than 1% of firms.

Firms with zero employees are assigned a size of 1. This is an attempt to deal with the ambiguity associated with incorporation. The owner of an incorporated sole-proprietorship is usually treated as an employee (by most statistical agencies), but the owner of an unincorporated sole-proprietorship is not. Both types of firms have a single member.

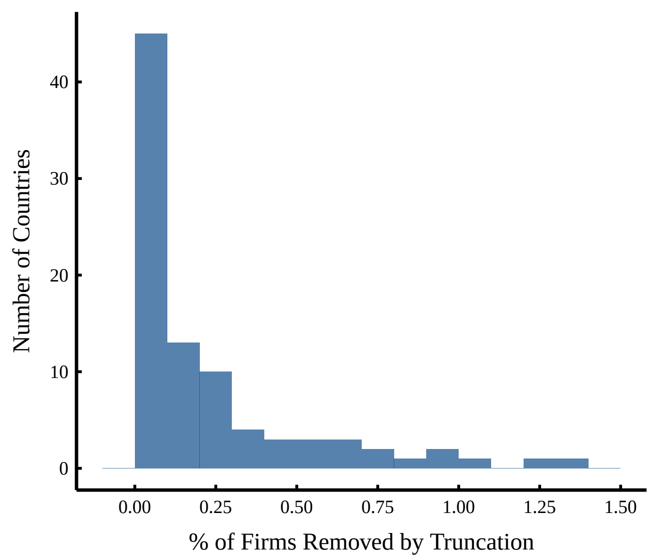

Figure 1: The Effects of Truncating the GEM Database $<$ 1000

This figure plots the country-level distribution of the percentage of firms removed by truncation (firms <1000). The x-axis shows the percentage of firms within each GEM country sample that are removed by truncation. The y-axis shows the number of countries with the given percentage range.

To compare the resulting firm size observations with other time-based series, I use the average year of each country's aggregated data.

Uncertainty in mean firm size is estimated using the bootstrap method [2]. This involves resampling (numerous times, with replacement) the data for each country and calculating the mean of each resample. Confidence intervals are then calculated using the resampled mean distribution.

For comparison between firm size and energy consumption, Yemen and Trinidad are removed as outliers.

Firm size – United States

Average firm size data for 1977-2013 is calculated by dividing the number of persons engaged in production (BEA Table 6.8B-D) by the number of firms. The latter is calculated as the sum of all employer firms in US Census Business Dynamics Statistics plus the number of unincorporated self-employed individuals (BLS series LNU02032192 + LNU02032185).

Average firm size data for 1890-1976 uses firm counts from HSUS Ch408 (which excludes agriculture) and total private, non-farm employment from HSUS Ba471-473 (total employment less farm and government employment). To construct a continuous time-series, the two data sets are spliced together at US Census levels for 1977.

Firm size – US Industry

Mean firm size is calculated using data from Statistics of U.S. Businesses, US 6 digit NAICS and 4 digit SIC between 1992 and 2013. 'Industry' is defined to include Mining, Construction and Manufacturing.

Firm size – US Manufacturing Sub-sectors

Mean firm size is calculated using data from Statistics of U.S. Businesses, US 6 digit NAICS 2002, 2006, and 2010.

Government Employment Share – International

International government employment data is from ILO LABORSTA database (total public sector employment: level of government = Total, sex code = A, sub-classification = 06). Total employment in each country uses World Bank series SL.TLF.TOTL.IN.

Government Employment Share – United States

US government employment data is from HSUS Ba473 (1890-1928), Ba1002 (1929-40), and BEA 6.8A-D persons engaged in production (1940-2011). Total US employment is from HSUS Ba471 (1900-1928), Ba988 (1929-1940), and BEA tables 6.8A-D (1941-2011).

Large Firm Employment Share – International

The measurement of the large firm employment share is inspired by the work of Nitzan and Bichler [4]. Global data is from Compustat Global Fundamentals (series EMP). Total employment in each country uses World Bank series SL.TLF.TOTL.IN. In some countries, the Compustat data exhibits sharp discontinuities. In order to remove these discontinuities, I have removed the following data: Thailand (1999, 2008, 2010, 2011), Phillipines (2003), Croatia (2011, 2012), and Oman (2010).

Large Firm Employment Share – United States

Data for the largest firms in the United States (ranked by employment) is from Compustat North America, series DATA29 (Figure 2 uses the top 200 firms, while Figure 3 uses the top 25). Total US employment is from BEA tables 6.8A-D (Persons Engaged in Production).

Large Firm Employment Share – US Industry

The employment of the largest 25 firms in US Industry is calculated using the Compustat database, series DATA29. 'Industry' is defined to include Mining, Construction, and Manufacturing (all SIC codes between 1000 and 3999). Total Industry employment is from BEA tables 6.8A-D (Persons Engaged in Production).

Large Firm Employment – US Manufacturing Subsectors

Large firm employment share is calculated using data from Statistics of U.S. Businesses, US 6 digit NAICS 2010. 'Large firms' are defined here as those with 5000 or more employees. This differs from other data in Figure 3 of the main paper, where the 25 largest firms are used. Figure 2 analyzes the bias in this method. As expected, the number of firms with 5000 or more employees varies significantly by manufacturing subsector. However the median value is 26 firms, meaning that this method should yield similar results to the 'top 25' method used elsewhere. There is also no significant correlation between the number of firms with 5000 or more employees, and the sectoral employment share of these firms. Therefore, the variability in the sample size of 'large firms' does not cause a directional bias to the employment share of 'large firms'.

Management Employment Share

Management fraction = management employment / total employment. International management employment is from the ILO LABORSTA database using ISCO-88 (Legislators, senior officials and managers) and ISCO-1968 (Administrative and managerial workers).

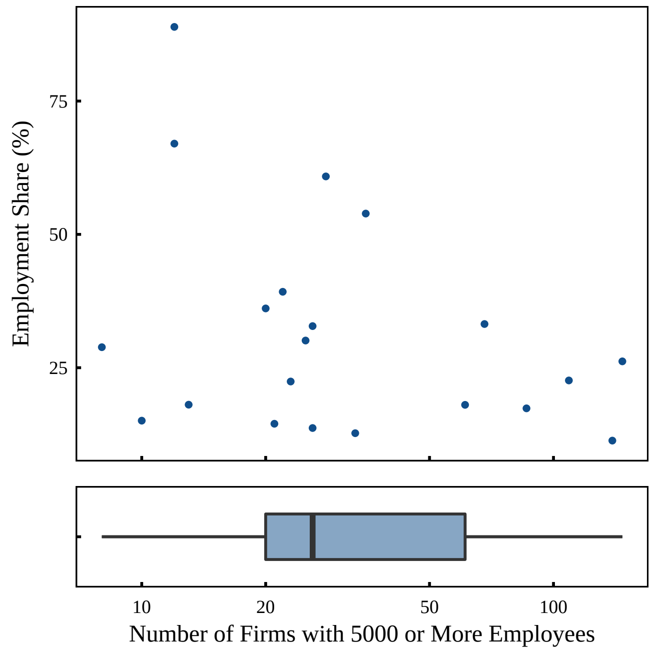

Figure 2: Large Firms in Manufacturing Subsectors — Analyzing Bias Caused by Variations in the Number of Firms

The top panel plots the employment share of 'large firms' versus the number of firms that are defined as 'large' ($\geq$ 5000 employees). Each data point represents a single manufacturing subsector. The bottom panel shows the distribution of the number of 'large firms' per subsector.

Total employment is from World Bank series SL.TLF.TOTL.IN. For ISCO-88, Argentina is removed as an outlier. For ISCO-1968, Syria is removed as an outlier.

US management employment is from BLS Occupational Employment Statistics (various tables, 1999-2014), ILO LABORSTA ISCO-88 (1970-1998) and HSUS Ba1037 (1860-1970). All series are spliced to BLS data. Total US employment is from HSUS Ba1033 (1860-1890), HSUS Ba471 (1900-1928), Ba988 (1929-1940), and BEA tables 6.8A-D (1941-2011). All series are spliced to BEA data.

Power Plants – Construction Labor Time vs. Capacity

Data is compiled by the author from numerous sources. Data and sources are provided in spreadsheet form in S1 File Data and Code.

Power Plants — US Plant Mean Capacity

Plant nameplate capacity data comes from EIA 860 forms from 1990 to 2015. Mean plant capacity counts only power plants that are operational in the given year. Note that form

860 reports *generator* capacity. To calculate *plant* capacity, I aggregate all generators with the same Plant Code.

Self-Employment — International

International self-employment data is from the World Bank, series SL.EMP.SELF.ZS.

Self-Employment — United States

US self-employment data is from HSUS Ba910 (1900-1928), Ba988 (1929-1940) and BEA tables 6.7A–D (1941-2011). Total US employment is from HSUS Ba471 (1900-1928), Ba988 (1929-1940), and BEA tables 6.8A-D (1941-2011).

Self-Employment — US Industry

Industry self-employment data is from BEA tables 6.7A–D. Industry total employment is from BEA tables 6.8A-D (Persons Engaged in Production). Industry is defined to include Mining, Construction, and Manufacturing.

Small Firms — US Manufacturing Subsectors

Small firms are defined as those with 0–4 employees. Data is from Statistics of U.S. Businesses, US 6 digit NAICS 2002, 2006, and 2010.

Span of Control

The span of control is calculated as the employment ratio between adjacent hierarchical levels. Data sources are listed in Table 1.

Technological Scale

Data for technological scale increases (shown in Table 1 of the main paper) is compiled by the author. Sources are available in spreadsheet form in the S1 File Data and Code.

Table 1: Span of Control Data Sources

Source	**Ref**	**Years**	**Type**	**N**	**Country**	**Firm Levels**
Ariga	[5]	1981-1989	A	unknown	Japan	All
Audas	[6]	1992	C	1	Britain	All
Baker	[7]	1969-1988	C	1	United States	Management
Bell	[8]	2001-2010	A	552	United Kingdom	Top 3
Dohmen	[9]	1987-1996	C	1	Netherlands	All
Eriksson	[10]	1992-1995	A	210	Denmark	Management
Heyman	[11]	1991,1995	A	560	Sweden	Management
Lima	[12]	1991-1995	C	1	Portugal	All
Morais	[13]	2007-2010	C	1	Undisclosed	All
Mueller	[14]	2004-2013	A	880	United Kingdom	All
Rajan	[15]	1986-1998	A	∼300	United States	Top 2
Treble	[16]	1989-1994	C	1	Britain	All

Notation: Ref = Reference, N = number of firms A = Aggregate Study, C = Case Study

Notes: The 'Firm Levels' column indicates the coverage of the study. 'All' indicates that the study covered all hierarchical levels with the firm(s). 'Management' indicates that only managers were studied. 'Top 2' and 'Top 3' indicate that only the top 2 or 3 hierarchical levels were studied. Raw data from Baker (the BGH dataset) is available for download at http://faculty.chicagobooth.edu/michael.gibbs/.

In many cases, the above papers report results in a table of values, which were then used in this paper. However, some papers report their results only in graphical form. In these cases, I used the *Engauge Digitizer* program to extract data from the graphics.

B Assessing Size Bias within Firm Databases

Like all scientific inquiry, the study of firm size distribution requires reliable data. Unfortunately, accurate firm-size data (with reasonable international coverage) is difficult to find. There are two primary data avenues available: government statistics (the *macro* level) and firm-level databases (the *micro* level). Each avenue has drawbacks.

The problem with relying on macro-level data is that it intrinsically limits the number of countries that can be studied. Apart from wealthy (OECD) nations, reliable macro statistics on firm size distribution are hard to find. This dearth of data often leads researchers to use micro-level databases instead.

The problem with using these micro-level databases to study firm size distribution is that they are rarely (if ever) designed to be *accurate* samples of the wider firm 'population'. As the analysis in this section demonstrates, firm-level databases typically under-represent small firms and over-represent large-firms. Thus, when using a micro database to study the firm size distribution, one must ask: is the database an accurate sample of the firm population? The question that immediately follows is: how do we know if the database is (or is not) biased?

In order to assess database bias, one must inevitably make comparisons to macro-level data. The key is to find macro data that is both relevant and *available* (the second criteria being the more difficult to fulfill). In the following sections I present and apply two methods for assessing firm-size bias within micro datasets.

Methods for Determining Firm-Size Bias within a Database

Method 1: Compare *macro* and *micro*-level average firm-sizes.

Method 2: Compare *micro*-level small-firm employment share to *macro*-level self-employment rates.

Method 1 is straightforward: it involves calculating the average firm-size within a *micro* database and comparing it to the average firm-size calculated from *macro* data. This approach is limited by the availability of macro data. For OECD countries, it is possible to directly compare firm-size averages between micro and macro data. I conduct such an analysis in Table 2 (visualized in Figure 6). Unfortunately, for most non-OECD countries, this approach is not feasible because relevant macro-level data does not exist (hence our need for micro data in the first place).

Method 2 is more indirect (and is dependent on some assumptions); however, its advantage is that self-employment data is readily available for most countries. The basic logic of method 2 is as follows:

1. Self-employed individuals work in *small* firms.
2. We can think of the self-employment rate as an indicator of the share of employment held by the smallest firms.

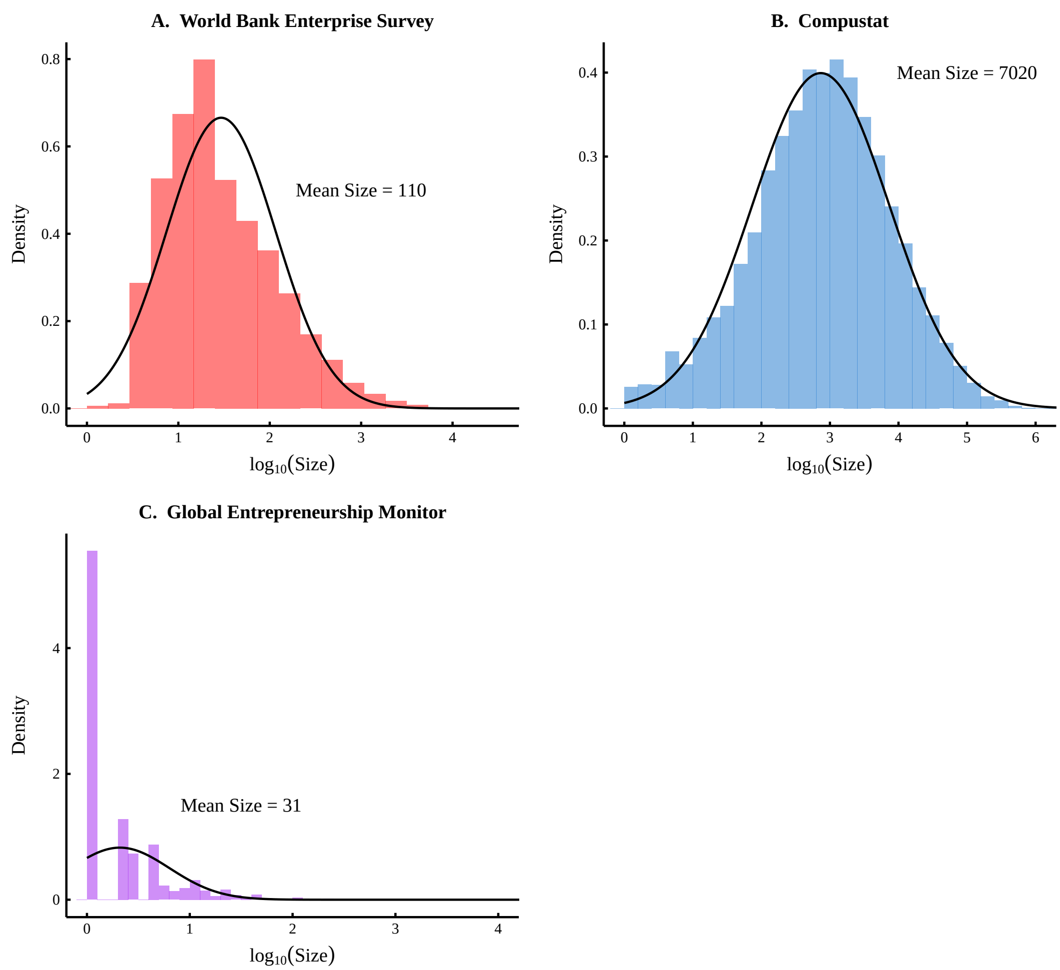

Figure 3: Firm Size Distributions in Selected Micro Databases

Histograms show the firm size distribution within each database (firm size = number of employees). Note that data is log-transformed. Black curves show the best log-normal fit. Panel A shows the firm size distribution of the entire World Bank Enterprise Survery database (for all years). Panel B shows the firm size distribution within the Compustat database (Compustat North America merged with Compustat Global – all available years). Panel C shows the firm size distribution of the Global Entrepreneurship Monitor (GEM) database (from 2000-2011). Note that the log-normal distribution fits both World Bank and Compustat data fairly well, but fits the GEM data very poorly.

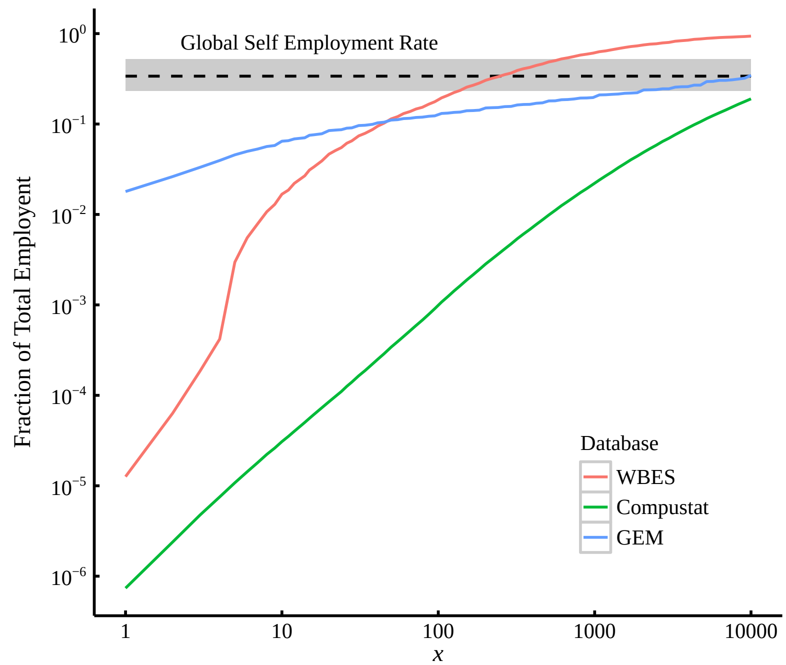

Figure 4: Small Firm Employment Share in Selected Micro Databases

This figure assesses the relative bias within the World Bank Enterprise Survey (WBES), Compustat, and Global Entrepreneurship Monitor (GEM) databases. The share of employment held by firms with x or fewer employees (in each database) is compared to the global self-employment rate between 1990 and 2013 (the dotted line is the median, while the shaded region shows the interquartile range). Sources: Global self-employment data is for self-employed workers who are *non-employers*. This is calculated by subtracting *employer* rates (series SL.EMP.MPYR.ZS) from total self-employment rates (series SL.EMP.SELF.ZS).

---

3. By comparing the self-employment rate to the small-firm employment share within a particular database, we can infer the degree of database bias.

As a starting point, I believe method 2 is more useful, since relevant data is more widely available. In Section B I apply method 2 to three databases: Compustat, the World Bank Enterprise Survey (WBES), and the Global Entrepreneurship Monitor (GEM). Figure 3 shows the firm size distribution within these three databases. The distributions are log-transformed in order to show the log-normal character of two of the three databases (Compustat and WBES).

While all three databases are global in scope, their respective firm size distributions are quite different (note the disparities in mean firm-size). Which database gives the most accurate picture of the underlying population of firms? Analysis reveals that the GEM database is the most consistent with available macro data. Based on these results, in Section B I then conduct a more detailed analysis of the GEM database (see Fig. 6).

Small Firm Employment Share as a Database Bias Test

The basic methodology of this test is to use macro-level self-employment rates as an indicator of the share of employment held by small firms. By comparing this rate to the small-firm employment share within a micro database, we can assess the level of bias.

To begin, we define the small firm employment share as the share of employment held by firms with x or fewer employees (where x is an arbitrary number). We then vary x and see if we can match the resulting small-firm employment share with empirical self-employment rates. Figure 4 conducts such an analysis on the Compustat, GEM, and WBES databases by comparing their respective small firm employment shares to the global self-employment rate.

First, we note that the small firm employment share in all three databases matches global self-employment rates only for a choice of x that is too large to be believably related to 'self-employment'. For WBES, the small firm employment share is similar to the global self-employment rate when x is of order 100. For the GEM and Compustat databases this does not happen until x is of order 10000. This suggests that all three databases have a significant bias towards the under-representation of small firms.

Which database has the least bias? To decide this, we must settle on a believable range for the size of self-employer firms. In the real-world, the boundary x, separating *self-employer* from *employer*, does not exist. However, we can make an educated guess at the likely size range of self-employer firms.

Although a firm size of 1 typically comes to mind when we think of self-employment, the statistical definition of 'self-employment' (as defined by the World Bank) is quite broad. It consists of the following sub-categories:[1]

1. Own-account workers
2. Members of producers' cooperatives
3. Contributing family workers

The inclusion of contributing family workers is important, especially in developing countries where household production is still common. In this context, the size of a self-employer 'firm' will be similar to the size of a family. Since very few families are larger than 10, a believable range for which the small firm employment share should relate to self-employment rates is for $1 \leq x \leq 10$.

Over this range, the GEM small firm employment share is by far the closest to the actual rate of self-employment. While the WBES claims to be a "representative sample of an economy's private sector", this analysis suggests otherwise. The WBES small firm employment share is 2-4 orders of magnitude off the global self-employment rate for $1 \leq x \leq 10$. The Compustat database produces even worse results (off by 4-5 orders of magnitude), but this is expected. Compustat maintains records only for *public* corporations, giving it an inherent bias towards larger firms.

[1] World Bank self-employment data also contains a fourth category called 'Employers'. This category is more aptly called 'owners'. Since firms of all size have owners, I have adjusted the self-employment rate by subtracting the 'Employer' rate.

Note that the WBES and GEM small firm employment shares *cross* at a firm size of roughly 50. Why? The WBES contains very few small firms (size 1-10) and too many *medium* size firms (size 10-50). The GEM database, on the other hand, contains many small firms, but seems to contain too many large firms (size $>$ 1000). This causes the crossing behaviour observed in Figure 4.

This analysis indicates that the GEM database is the most consistent with observed global levels of self-employment. However, it still seems to contain some size bias. The problem, as I discuss in the next section, is that the GEM database contains too many extremely large firms.

Assessing Firm-Size Bias Within the GEM Database

While sufficient to weed out extremely biased databases, the method used in Figure 4 ignores the internal distribution of data within each database. In general, micro databases with global coverage do not contain equal sized samples for each country. Thus, a large, biased sample from one country could potentially skew the entire database, even if other samples are relatively unbiased. To further test database bias, it is important to group data at the national level. In this section I investigate national-level bias within the GEM database.

I begin with a continuation of the self-employment/small-firm method developed above. However, I now group all data at the national level. The GEM database contains firm samples from a total of 89 countries, 72 of which also have data available in the WDI database. For each country, the employment-share of firms with 5 or fewer employees is calculated (from GEM data) and compared to the WDI self-employed rate (non-employers only). This calculation is done for both the full GEM dataset, and a *truncated* version in which all firms with more than 1000 employees are excluded. This truncated version is tested on the hunch that the full GEM database still over-represents large firms (a hunch that is confirmed in Fig. 6).

The results of this analysis are shown in Figure 5. Both the full and truncated GEM databases have a small-firm employment-share distribution that is roughly equivalent to the WDI self-employment rate distribution. Of particular interest is the fact that the small-firm employment share within the truncated GEM database gives a nearly one-to-one prediction of WDI self-employment rates (see Fig 5A).

This analysis suggests that both the full and truncated GEM databases give a reasonably accurate sample of the international firm size distribution. In order to differentiate between the two, it is helpful to compare mean firm-size estimates with macro data. Due to macro data constraints, this must be done with a much smaller sample size than the 72 countries used above. Table 2 shows the 23 countries for which data is available.

Note that macro-level mean-size estimates are predicated on a few assumptions. Government published statistics usually include firm-counts for *employer* firms only (i.e. firms with employees). Non-employer firms are excluded. Thus, unincorporated self-employed individuals are typically not counted as 'firms' (incorporated self-employed workers are technically counted as employees of their business, and are thus employer firms). As a result, calculations done using official firm-counts only will give a mean firm-size that is

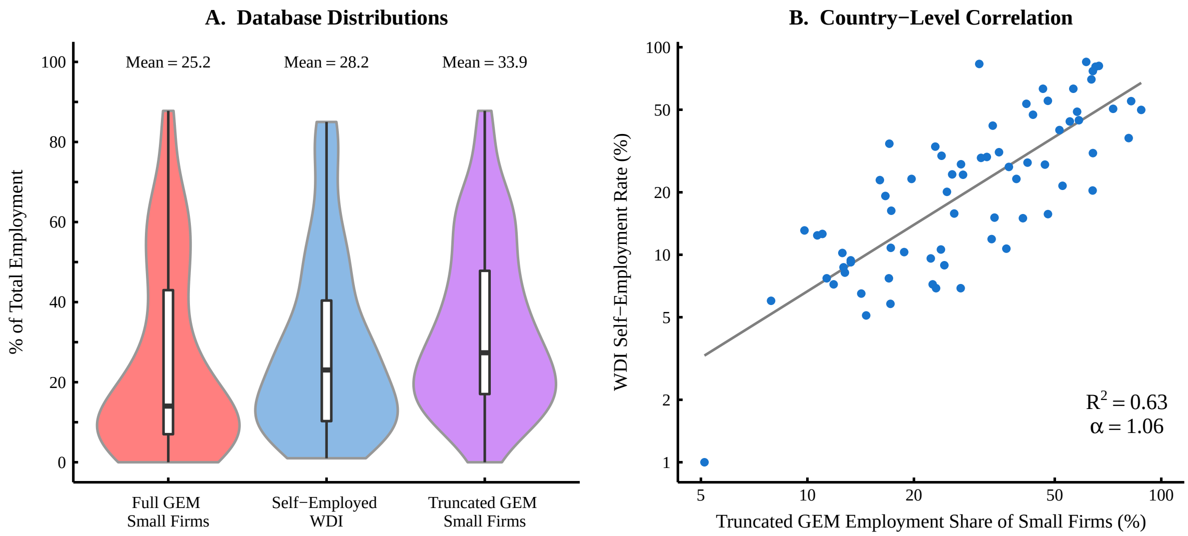

Figure 5: Assessing Small-Firm Bias in the GEM Database

Notes: This figure compares the employment share of small firms ($\leq$ 5 members) in the GEM database to the distribution of self-employment rates (non-employer firms only) within the WDI dataset. Only countries for which data is mutually available are shown (72 countries in total). Unlike Figure 4 all data is aggregated at the national level (countries with small/large sample sizes are all weighted equally). Panel A shows how country-level data is distributed within each database. The 'violin' shows the distribution of data. The internal box plot shows the interquartile range (the 25th to 75th percentile), with the median marked as a horizontal line. Corresponding mean values are shown above. Panel B shows a scatter-plot of country-level data (each point is a country) for the self-employment rate vs. the small-firm employment share in the truncated GEM database. The line shows the best-fit power regression. Note that the regression exponent, α, is nearly 1. Thus, the relation between self-employment rates and small-firm employment share is roughly one-to-one. A similar regression for the non-truncated GEM database (not shown) gives $R^2 = 0.48$ and $\alpha = 0.54$, far from a one-to-one relation. This discrepancy between the full and truncated GEM dataset is the result of the over-representation of large firms within a handful of countries. This skews the small firm employment share downwards (note the low median for the full GEM database in Panel A). Thus, the truncated GEM database is more consistent with self-employment data, meaning we can infer that it has less of a firm-size bias.

Sources: Non-employer rates are calculated by subtracting employer rates (series SL.EMP.MPYR.ZS) from the total self-employment rate (series SL.EMP.SELF.ZS). WDI data is chosen for which the data year most closely matches the GEM year (which is calculated as the country-level mean year of all data entries from 2000-2011).

Table 2: Mean Firm-Size in the GEM Dataset vs. Macro Data

Country	Macro	GEM Trunc	GEM Full
Austria	7.6	11.7	12
Belgium	5.6	6.3	6
Czech Republic	3.5	13.5	30
Denmark	9.2	8.5	26
Finland	6.8	5.3	13
France	7.5	5.3	22
Germany	10.4	11.9	151
Hungary	5.7	6.1	8
Italy	3.5	2.8	17
Netherlands	6.1	10	27
Poland	4.7	2.9	16
Portugal	3.6	8.9	9
Russian Federation	18	9	16
Slovakia	4.2	11.8	17
Slovenia	4.9	13	19
Spain	5.5	4.5	10
Sweden	5.8	5.7	15
Switzerland	10.8	6.5	180
Turkey	3	9.5	18
United Kingdom	7.7	7	26
United States of America	9.1	10	164
India	2.6	5.2	6
Ghana	1.5	2.2	2
Mean	6.4	7.7	35.2

Notes: This table compares mean firm sizes within the GEM database to macro-level data. Data is shown for both the full GEM database, and its truncated version, which removes all observations of firms with more than 1000 employees. The rational for truncation is that large firms are over-represented within the dataset, skewing mean firm-size.

Sources and Methodology: Macro-level mean firm-size is calculated by dividing total employment by the number of firms. The number of firms N_{total} is calculated using Eq. (1), where N_{gov} is government data for the number of firms, S_T is the self-employment rate, S_E the self-employed *employer* rate, U is the fraction of self-employed firms that are unincorporated (hence not counted in official statistics), and L is the size of the labor-force.

$$N_{total} = N_{gov} + (S_T - S_E) \cdot U \cdot L \tag{1}$$

Data for S_T, S_E and L come from World Development Indicators (WDI) series SL.EMP.SELF.ZS, SL.EMP.MPYR.ZS, and SL.TLF.TOTL.IN, respectively. Data for the official number of firms comes from OECD Entrepreneurship at a Glance 2013. Due to lack of data, U is assumed to be 0.7, the level observed in the US [17]. For Ghana, all data comes from Sandefur [18], Table 1 and 2. For India, all data comes from Hasan and Jandoc [19], Table 1 and Table 3 (using the sum of the ASI and NSSO datasets). For US data sources, see Appendix A.

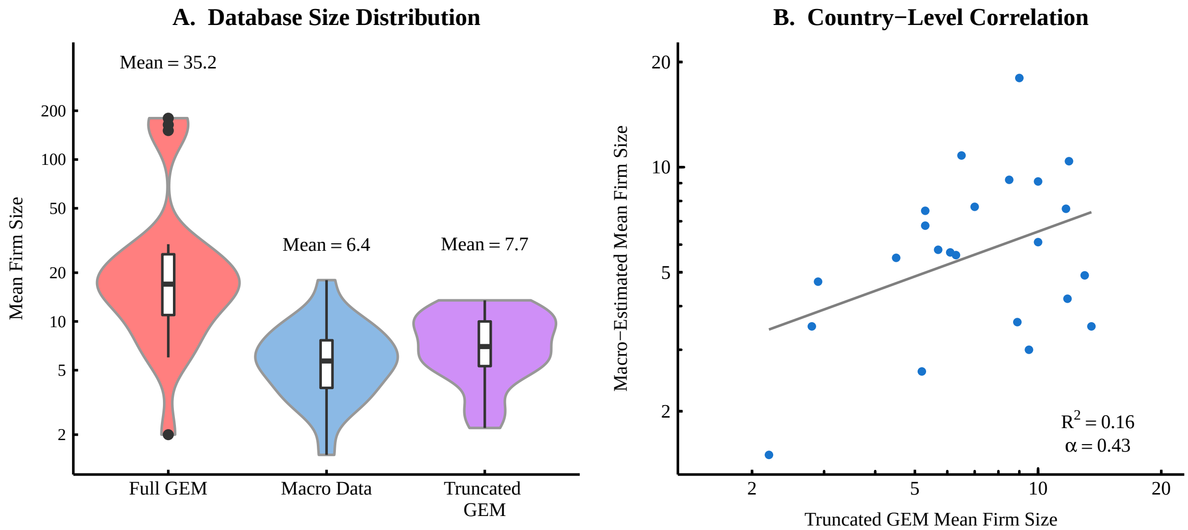

Figure 6: GEM Mean firm size distribution vs. Macro Data

Notes: This figure visualizes the mean firm-size data for the countries shown in Table 2. Panel A shows the mean firm-size distribution within each database. Relative to macro data, the full GEM database clearly over-represents large firms. The mean firm-size in the truncated GEM database is also slightly larger than the macro data, but given the small sample size, the difference is statistically insignificant ($p = 0.20$). Panel B shows the correlation between macro data and the truncated GEM data. A power regression gives an exponent $\alpha = 0.47$, below the desired one-to-one level that would indicate perfect agreement between the micro and macro data. Despite these shortcomings, the truncated GEM database appears to be a fairly accurate sample of the international firm-size distribution.

disproportionately large. To account for this bias in macro data, I adjust the official firm-count by adding an estimate for the number of self-employer firms (see the methodology in Table 2).

The results of this investigation are visualized in Figure 6. From this analysis, there is convincing evidence that the full GEM database over-represents large firms. For a few countries (Germany, Switzerland, and the US) this leads to a mean firm-size estimate that is a factor of 10 larger than macro estimates. Truncating the GEM database seems to effectively adjust for this bias.

Why is truncation effective (and is it justified)? The problem of firm-size bias is partially due to the extremely skewed nature of the firm size distribution. The presence of even a *single* extremely large firm can have a large effect on the mean of a sample. For instance, the GEM database contains roughly 170,000 observations. Suppose that the mean firm-size of these observations is 5. If we add a single observation of a Walmart-sized firm (2 million employees), the resulting average more than *triples* (to roughly 17). Of course, firms this large do exist, but the chance of observing one in a sample should be *extremely* small.

The fact that large firms are over-represented in the GEM database demonstrates a sampling bias. Discarding observations of very large firms is one method for dealing with this bias. Other methods are certainly possible, but I do not discuss them here.

Functional Form of the Firm Size Distribution

One of the first tasks for understanding an empirical distribution (of any kind) is to look for theoretical distributions that can be used to model it. Many observers have used the log-normal distribution to model firm size distributions [20, 21, 22, 23, 24, 25]. As shown in Figure 3, the log-normal distribution is a suitable model for the firm size distribution within the Compustat and WBES databases. However, the preceding analysis showed that these databases are rather poor representations of the actual global firm size distribution.

It may be that the use of the log-normal distribution is an artefact of researchers' reliance on biased micro databases [26]. For data that is more representative of the actual firm size distribution (i.e. the GEM dataset), a power law distribution is a much better fit. The characteristic feature of the log-normal distribution is that its logarithm is *normally distributed* (hence the reason for the log transformation in Fig. 3). A power law distribution, however, will not *not* appear normally distributed under a log transformation. Instead, it will decline monotonically as the GEM database does.

Unlike Compustat and WBES, the GEM database is much better fitted with a power law than with a log-normal distribution (see Fig. 7A). For firms under 10,000 employees, the GEM database is consistent with a power law with a scaling exponent $\alpha \approx 1.9$. Note that the tail of the GEM database is 'fatter' than expect for a power law (it is above the 99% confidence interval). This is consistent with our earlier conclusion that the GEM database over-represents large firms. Macro data from for the US firm size distribution is also consistent with a power law (Fig. 7B).

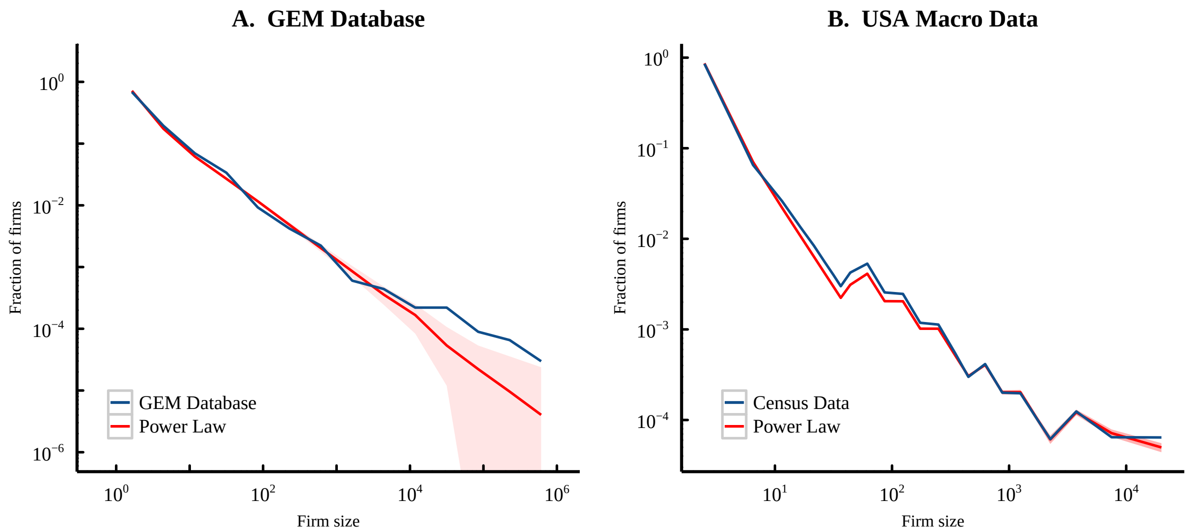

Figure 7: GEM and US Census Data are Consistent with Power Laws

Notes: Panel A shows the firm size distribution of the Global Entrepreneurship Monitor database (all years). For firms with less than 10,000 employees, the database is consistent with a discrete power-law distribution with exponent $\alpha \approx 1.9$. Panel B shows the US firm size distribution, which is consistent with a discrete power-law distribution with exponent $\alpha \approx 2$. Shaded regions show the 99% confidence interval for a simulated power law distribution with a sample size similar to each dataset.

Sources and Methodology: US data for employer firms is from the US Census Bureau, Statistics of U.S. Businesses (using data for 2013). This data is augmented with Bureau of Labor Statistics data for unincorporated self-employed workers (series LNU02032185 and LNU02032192). The histogram preserves Census firm-size bins, with self-employed data added to the first bin. The last point on the histogram consists of all firms with more than 10,000 employees. Both power-law distributions are simulated using the R poweRlaw package, and plotted with the same histogram bins used to plot empirical data. The GEM simulation uses 170,000 observations while the US simulation use 10 million observations.

Note: many readers will expect power law distributions to appear linear when plotted on a log–log scale. Departures from linearity shown in Panel B are artefacts of US census bin sizes (which do not always grow proportionately).

C The Firm Size Distribution as a Variable Power Law

Recent studies have found that firm size distributions in the United States [26] and other G7 countries [27] can be modelled accurately with a power law. Less is known about other countries. In this section, I test if country-level firm size distributions in the GEM database are consistent with a power law. I find that a power law distribution is favored over other heavy-tail distributions in the vast majority of countries. I also find that international variations in 3 summary statistics (mean, self-employment, and large firm employment share) are mostly consistent with a power law distribution.

Power Laws in the GEM Database

The firm size distribution in the *entire* GEM database is roughly consistent with a power law, although the end of the tail is slightly too heavy (Fig. 7A). In this section, I analyse the GEM firm size distribution at the *country* level to assess how well the data fit a power law distribution. I use the truncated GEM database, which contains only firms with fewer than 1000 employees. The rational is that the full GEM database slightly over-represents large firms (see Appendix B).

Historically, power law distributions have been fitted by using an ordinary least-squares (OLS) regression on the logarithm of the histogram. However, this approach is inaccurate, and it violates the assumptions that justify the use of OLS [28]. A more appropriate approach for fitting distributions is to use the *maximum likelihood method*. The likelihood function $\mathcal{L}$ assesses the probability that a set of data x came from a probability density function with the parameter(s) θ.

$$\mathcal{L}(\theta|x) = P(x|\theta) \tag{2}$$

The best fit parameter(s) θ_{mle} maximizes the likelihood function. Like any fitting method, the maximum likelihood indicates only the best fit parameters of the *specified* model, not the appropriateness of the model itself. To discriminate between two different models (1 and 2), we compare their respective maximum likelihoods in ratio form (Λ). The larger likelihood indicates the better fitting model.

$$\Lambda_{1,2} = \frac{\mathcal{L}_1(\theta_{mle}|x)}{\mathcal{L}_2(\theta_{mle}|x)} \tag{3}$$

It is often more convenient to use the log-likelihood ratio, $\log \Lambda$. The sign of $\log \Lambda$ indicates the preferred model (positive indicates that model 1 is better, negative indicates that model 2 is better). The magnitude of $\log \Lambda$ indicates the strength of this preference.

I use this method to assess if country-level firm size distributions in the GEM database are best modelled with a power law. I compare the likelihood of a power law distribution to the likelihood of four other heavy-tail distributions: gamma, log-logistic, log-normal, and Weibull. The resulting range of log-likelihood ratios (one for each country in the GEM database) is shown in Figure 8. A power law distribution is favored over other distributions in the vast majority of countries (97%).

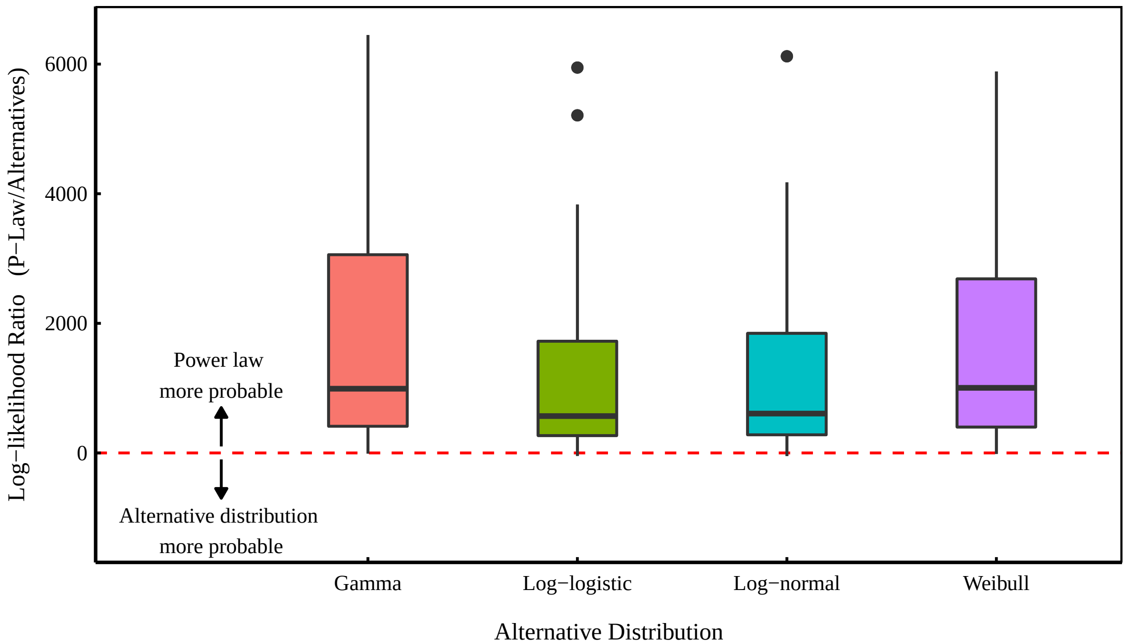

Figure 8: Comparing the Power Law to Alternatives in the GEM Database

Using country-level firm size distributions from the GEM database, this figure assesses the goodness of fit of a power law relative to four other heavy-tail distributions. The firm size distribution in each country in the GEM database is fitted with a power law, gamma, log-logistic, log-normal, and Weibull distribution. For each country, the log-likelihood ratio is computed between the power law and the four alternative distributions. The box plots display the resulting range of ratios. A positive ratio indicates that the power law is more probable, while a negative ratio indicates that the alternative distribution is more probable. In order to better display the majority of data, several large outliers favoring a power law are not shown. For all but 3 countries, a power law distribution is the best fit.

Notes: This figure shows the mean log-likelihood ratios for 100 re-samples (with replacement) of each country. Maximum likelihoods are calculated using the R packages 'poweRlaw' (for a power law) and 'fitdistrplus' (for alternative distributions). Although empirical data is discrete, all models used here are continuous.

International Summary Statistics

Firm size summary statistics can be used as another way to test if the firm size distribution is consistent with a power law. This has the advantage of broadening the evidence to include more data sources (I combine GEM, World Bank, and Compustat data). My method is to pair two statistics and test if the resulting empirical relation can be reproduced by simulated samples from a power law distribution. I look at two pairings: (1) the self-employment rate vs. mean firm size; (2) the large firm employment share vs. mean firm size.

Self-Employment vs. Mean Firm Size

The rational for looking at the self-employment rate is that it indicates the relative share of employment held by small firms. Figure 9A shows the empirical relation between self-employment rates and mean firm size (black dots). The simulated relation is shown in the background, where the power law exponent α is indicated by color. Creating this simulation requires making assumptions about the size of self-employer firms. I assume that *all* firms below the size boundary L_s are considered self-employer firms. The simulated self-employment rate then consists of the fraction of employment held by firms with employment less than or equal to L_s.

To account for international variation in the size of self-employer firms, I let the boundary point vary randomly over the range $1 \leq L_s \leq 10$. In Figure 9A, $L_s = 1$ corresponds to the bottom of the coloured region, and $L_s = 10$ to the top. Why choose the upper bound to be so large? My reasoning is based on the definition of 'self-employment', which consists of 3 sub-categories: own-account workers, cooperatives, and family workers.[2] Especially in developing countries, where household production is still common, a self-employer 'firm' is synonymous with a *family*. A size of 10 seems a reasonable upper limit on the size of family. Given this assumption, a majority of countries (75%), as well as the entire time series for the United States, have a self-employment vs. mean firm size relation that is consistent with a power law.

Large Firm Employment Share vs. Mean Firm Size

To test if variations in the large firm employment share are consistent with a power law distribution, I use the same method as above: I plot the employment share of the 100 largest firms against mean firm size (Fig 9C). I then compare this relation to the one predicted by simulated power law data. To allow for the effects of differing country size, simulation sample sizes vary over the range of national firm populations (which are estimated by dividing the labor force by the mean firm size).

A slight majority of countries (56%), as well as the entire time-series for the United States, have a large firm employment share vs. mean firm size relation that is consistent with a power law distribution. Note that all data points that are *not* consistent with a power

[2] Most statistical databases add a fourth category of 'employers' (i.e. capitalists). Because this category is not related to small firms, I remove it from analysis.

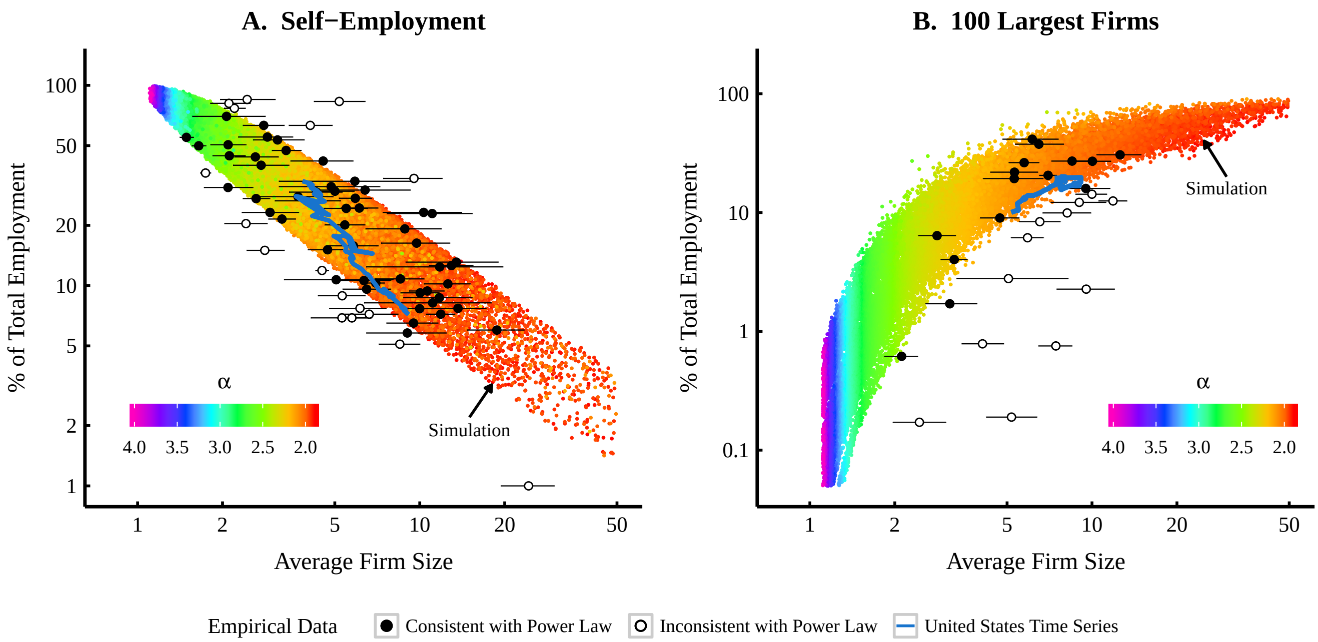

Figure 9: International Summary Statistics, Empirical vs. Power Law

This figure compares pairings of summary statistics for empirical and simulated data. Empirical data is at the country level. Simulated data is randomly generated from a power law distribution (the exponent α is indicated by color). Panel A shows self-employment rates vs mean firm size while panel B shows large firm employment share vs. mean firm size. Self-employment rates are modelled as the employment share of all firms less than the size L_s, which varies randomly over the range $1 \leq L_s \leq 10$. Uncertainty in mean firm size (95% level confidence intervals) is indicated by horizontal lines. Empirical data is judged to be consistent with a power law when the error bar is within the 99% range of simulated data. For data sources, see Appendix A.

---

law lie *below* the simulation zone (rather than above). This could indicate that these countries have firm size distributions with a tail that is thinner than a power law, but it could also indicate a problem with the data. I have assumed that the 100 largest firms in the Compustat database are actually the largest firms in each nation. There is no guarantee that this assumption is true: the Compustat database may not give complete coverage of the largest firms, especially if a country has many large *private* companies. Further research is needed to determine if these findings indicate a departure from a power law distribution, or if they are artefacts of incomplete data.

D Testing Gibrat's Law Using the Compustat Database

Gibrat's 'law' states that firm growth rates are *independent* of firm size. To what extent is this supported by empirical evidence? I investigate here using the Compustat US database. My results are consistent with previous analysis of the Compustat database: growth rates are approximately Laplace distributed, and volatility declines with firm size [29]. However, I show that this decline is of importance to only a small subset of firms.

Analysis

Rather than directly calculate the mean and variance of Compustat firm growth rates, I fit the growth rate distribution with a truncated Laplace density function (growth rates less than -100% are rounded to -100%). I then investigate how the parameters of this function change with firm size (Fig. 10). The advantage of this approach is that it is not biased by large outliers, and it allows a direct comparison of empirical data to modelled data (where firm growth rates are drawn from a Laplace distribution).

To estimate the Laplace parameters, I fit the histogram of simulated data to the histogram of empirical data (using a Monte Carlo technique that minimizes the absolute value of the error). The results are displayed in Figure 10C-D. The location parameter (μ) shows no significant relation to firm size. However, growth rate volatility (the scale parameter, b) declines monotonically with firm size.

Interestingly, the location parameter is always less than zero, meaning the most probable rate of growth is *negative*. This finding is consistent with the conditions predicted by a stochastic model with a reflective lower bound. Such a model will be stable only when there is a net *negative* drift to firm size (Appendix E). In Appendix F I reproduce the US firm size distribution using a model with a location parameter of -1%, which is consistent with Compustat data.

Extrapolating to the Entire Economy

Because the Compustat database contains data only for publicly traded firms, it is not an accurate sample of the wider US firm population (see Appendix B). However, based on the assumption that the US firm size distribution is a power law, we can estimate how the volatility-percentile relation shown in Figure 10D might look for the economy as a whole. The method for this process is shown in Table 3.

The first step is to generate a US firm sample from a power law distribution that best fits empirical data (I use $\alpha = 2.01$ here), and then compute size percentiles. Next, we select a particular percentile (the green cell) and note the corresponding firm size in the Compustat database (left pink cell). We than find all firms within the power-law sample that have the same size (right pink cells). The scale parameter for the selected Compustat percentile (left purple cell) is then mapped onto these firms, and their corresponding percentiles. The result (right purple cells) is a transformed relation between firm percentile and scale parameter that serves as our economy-wide estimate.

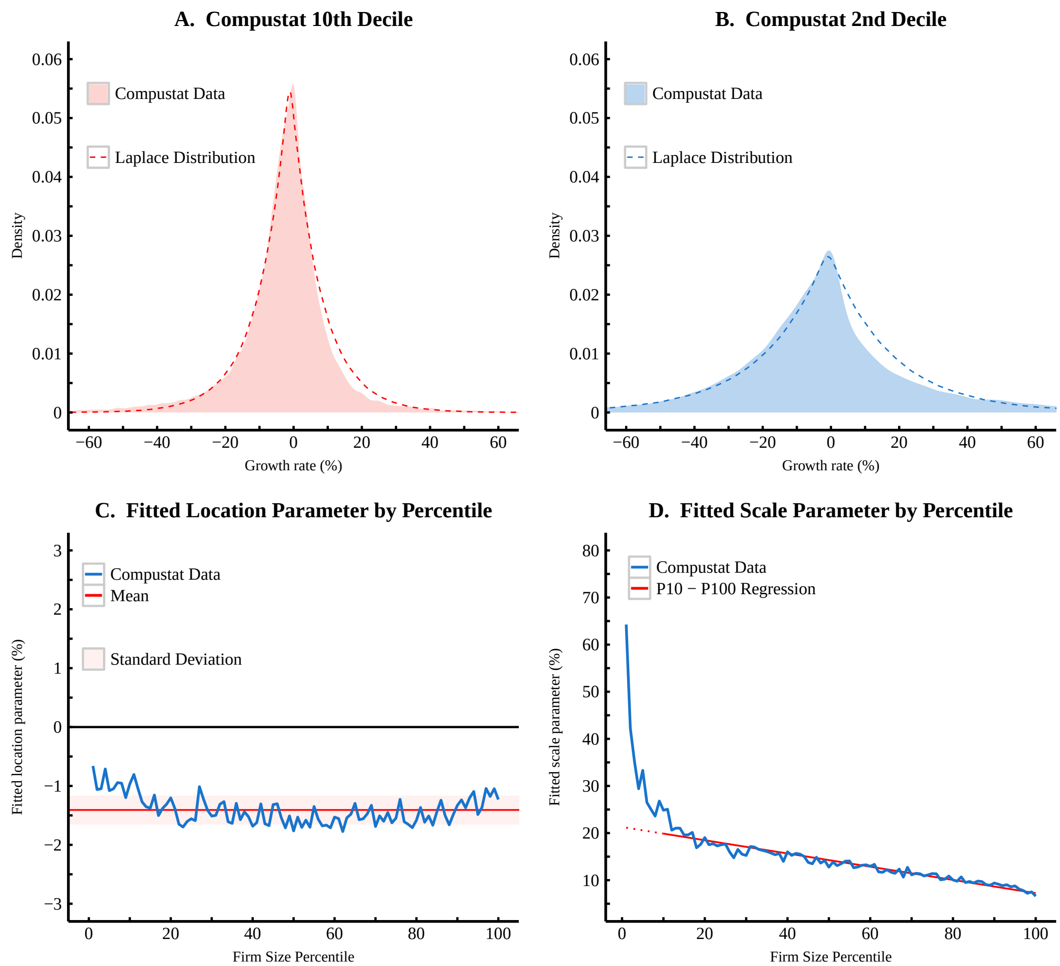

Figure 10: Firm Growth Rate Distribution in the Compustat US Database

This figure analyses firm growth rates (by employment) within the Compustat US database from 1970 to 2013. Panel A shows the growth rate distribution for firms in the 10th (top) decile, while Panel B shows the distribution for firms in the 2nd decile. Dotted lines indicate the best-fit Laplace distribution. Panel C and D show the results of Laplace regressions at the percentile level. Panel C shows the estimated location parameter (μ), while Panel D shows the estimated scale parameter (b). Laplace distributions are fitted using a Monte Carlo method. This analysis indicates that growth rate volatility is a function of firm size, while the growth rate mode is not. Given the firm-size bias of the Compustat database, results for lower percentiles (i.e. P1-P10) should be treated with scepticism.

Table 3: Method for Transforming Compustat Scale Parameter Regressions

Percentile	Compustat Firm Size	Scale	Power Law Firm Size	Transformed Scale
1	1	60	1	60
2	3	50	1	60
3	–	–	1	60
4	–	–	1	60
5	–	–	1	60
6	–	–	2	50

This table demonstrates the method for transforming the Compustat scale-percentile relation to an estimated relation for the whole economy. The first step is to select a percentile (the green cell P1 is selected here). We then match the Compustat firm size of this percentile to the equivalent power law firm size (pink cells). The Compustat scale parameter is then mapped onto all power law percentiles with matching firm sizes, resulting in a transformed scale function (purple cells).

The results of this transformation are shown in Figure 11. Two different estimates are shown. The blue curve shows results using the raw data shown in Figure 10D, while the red dotted curve shows results using a linear regression for P10-100, extrapolated over all percentiles.

Why two different methods? The bias in the Compustat increases as firm size decreases: coverage for large firms is nearly complete, while coverage of small firms (under 10) is extremely limited. Thus, it is quite possible that the large increase in volatility for firm percentiles 1–10 may be an artefact of this bias. By using the linear regression of P10-P100, we remove this potential artefact. We can think of the two curves in Figure 11 as representing a plausible range for the US economy. The stochastic model used to reproduce the US firm size distribution (Fig. 12), has a location parameter of 34%, which is much nearer the lower bound of our Compustat estimates.

This analysis suggest that declines in growth rate volatility are important only to a small minority of firms.

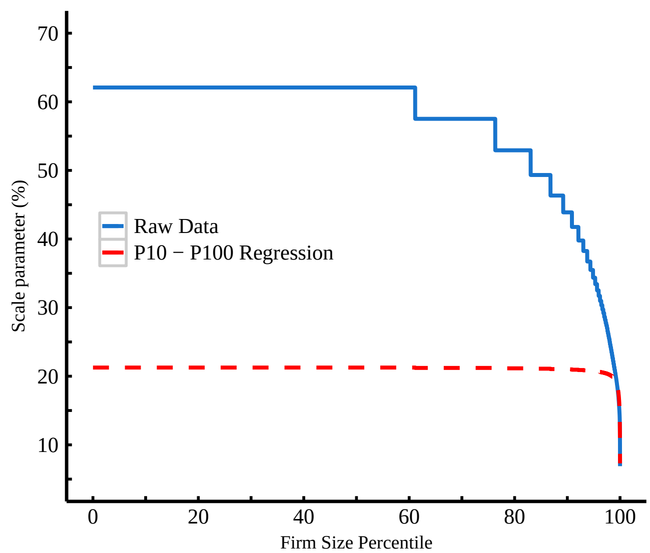

Figure 11: Scale Parameter vs. Percentile, Economy-Wide Estimates

This figure shows a transformation of the Compustat scale-percentile regressions (Fig. 10D) to a form that is consistent with the firm size distribution of the entire US economy. The US distribution is modelled with a power law ($\alpha = 2.01$). The blue curve shows the relation that would result from using the entire range of the Compustat regressions (P1-100). The step-wise pattern is a result of discrete data (steps correspond to a change in firm size by 1). The red dotted curve shows the relation resulting from using a linear regression of Compustat P10-100 (red line in Fig. 10D), extrapolated over P1-10.

E Instability of the Gibrat Model

The Gibrat model assumes that firm growth is a stochastic, multiplicative process. If L_0 is the initial firm size and x_i the annual growth rate, then firm size at time t is given by:

$$L(t) = L_0 \cdot x_1 \cdot x_2 \cdot ... \cdot x_t \;\; = \;\; L_0 \prod_{i=1}^{t} x_i \tag{4}$$

The instability of this model was first noted by Kalecki [30]. It stems from the model's diffusive nature: the resulting firm size distribution tends to *spread* with time. This tendency can be understood by relating the model to the classic example of diffusion: the one-dimensional random walk.

In a random walk model, a particle is subjected to a series of random additive shocks (y_i) that cause its position to change over time. At any given time, the particle's displacement from the initial position $d(t)$ is simply the sum of all of these shocks:

$$d(t) = y_1 + y_2 + ... + y_t \;\; = \;\; \sum_{i=1}^{t} y_i \tag{5}$$

In order to intuitively understand how this leads to diffusion, let us suppose that the shocks y_i are drawn from the uniform distribution $\{-1, 1\}$. At any given time, we can ask: what is the maximum possible displacement? In this case, it is exactly equal to t (the number of time intervals that have passed). When we introduce many randomly moving particles, some may attain this maximum displacement (however unlikely it is). Since the maximum grows with time, we can conclude that the displacement distribution must spread with time.[3]

The Gibrat model shares this property, except that the diffusion is *exponential*. To see this, we take the logarithm of Eq. 4, which allows us to express the growth rate product as a *sum*.

$$\begin{aligned}\log(L(t)) &= \log(L_0) + \log(x_1) + \log(x_2) + ... + \log(x_t) \\ &= \log(L_0) + \sum_{i=1}^{t} \log(x_i)\end{aligned} \tag{6}$$

We then exponentiate to get:

$$L(t) = L_0 e^{\sum_{i=1}^{t} \log(x_i)} \tag{7}$$

By setting $\log(x_i) = y_i$, we can see that Eq. 7 is just Eq. 5 in exponential form: our firm growth model is a one-dimensional, *exponential* random walk. The resulting firm size distribution will therefore spread rapidly with time – a fact that is inconsistent with

[3] For a step size drawn from the uniform distribution $\{-1, 1\}$, the standard deviation of the displacement is equal to $\sqrt{t}$. For a good derivation, see Feynman [31] Ch. 6.

available evidence. For instance, we know that the US firm size distribution has changed little since 1970 (see Fig. 2 in main article).

The second problem with this model is that it gives rise to a *log-normal* distribution, contradicting our finding that most firm size distributions are best described by a power law. The proof that this model leads to a log-normal distribution is straightforward. For a sufficiently large number of iterations, the Central Limit Theorem dictates that the sum of independent, random numbers will be *normally* distributed. Thus, for a large number of random walkers, the displacement $d(t)$ will be normally distributed (so long as the distribution of y_i satisfies certain conditions). Because Eq. 7 is the exponential form of Eq. 5, the *logarithm* of $L(t)$ will be normally distributed – the defining feature of the log-normal distribution.

Adding a Reflective Lower Bound

One simple way to reform this model is to add a *reflective lower bound* that stops firms from shrinking below a certain size [32, 33, 34]). This slight change will cause the model to generate a power law, rather than a log-normal distribution. It also leads to model stability (under certain conditions).

Why does the introduction of a reflective boundary lead to a power law distribution? One way of understanding this is to relate back to the additive random walk. If a reflective barrier is added to a one-dimensional random walk, it will no longer tend towards normal distribution; rather, it will tend towards an *exponential* distribution (see [35], p 15 for a proof).

Recall that a multiplicative process can be transformed into an additive process by taking the logarithm. Therefore, for a multiplicative firm model with a lower bound, the logarithm of firm size (L) will be exponentially distributed. Thus, the firm size distribution $p(L)$ is given by Eq. 8, which reduces to a power law (where C is the normalizing constant, and α is the scale parameter).

$$\begin{aligned} p(L) &= Ce^{-\alpha \cdot \log(L)} \\ &= CL^{-\alpha} \end{aligned} \tag{8}$$

For a firm size distribution, the obvious choice for a minimum lower bound is $L = 1$ (a sole-proprietor with no employees). In the proceeding model, I implement this reflection through the following conditional statement, which is evaluated at every time interval:

$$\text{if} \;\; L(t) < 1, \;\; \text{then} \;\; L(t) = 1 \tag{9}$$

Introducing a reflective lower bound also solves the instability problem, but only when growth rates have a negative 'drift'. Why? Intuitively, we can state that a model will be stable if it is not possible for a firm to shrink or grow *forever*. Introducing a lower bound automatically stops firms from shrinking forever, but it does nothing to stop the possibility of unending growth.

However, if firm growth rates have a net *downward* drift, all firms will tend towards a size of 0, given enough time. This downward drift occurs when the *geometric* mean of the growth rate distribution is less than 1. We can draw an analogy with gas particles moving in a gravitational field on earth. The particles move randomly, but there must be a small net downward drift due to the force of gravity. The result is a stable distribution of particles. If we remove gravity, the particles are free to diffuse forever. Similarly, if we remove the downward bias to firm growth rates, the distribution becomes unstable.

F Properties of Stochastic Models

Despite their simplicity, stochastic models of firm growth are able to replicate many important properties of the real world. I review three such properties here. Stochastic models can be used to:

1. Generate a firm size distribution that is consistent with empirical data;
2. Reproduce the relation between firm size and firm age;
3. Simulate new firm survival rates over time.

Modelling the US Firm Size Distribution

The model used here assumes scale-free growth with a reflective lower bound at a firm size of one. Growth rates are drawn from a Laplace distribution that is truncated by rounding all (fractional form) growth rates less than 0 to 0. In order to maintain a discrete distribution, firms with non-integer size are rounded to the nearest integer (after the application of each growth rate).

This simple model can be used to replicate the US firm size distribution (Fig. 12). In this case, model parameters $\mu = 0.99$ and $b = 0.34$ are used. The model shows the distribution of 1 million firms after 100 time iterations. In order to capture fluctuations around the equilibrium, the model is run 100 times, with the shaded region showing the resulting range of outcomes.

Firm Age vs. Firm Size

Firm age is calculated as the time since a firm's last 'reflection'. The model described above can be used to replicate the size-age relation of firms in the World Bank Enterprise Survey (WBES) database (Fig. 13A). The fitted parameters are $\mu = 0.97, \;\; b = 0.55$. Note that the model diverges from WBES data for firms with fewer than 10 employees. Due to the size bias within the WBES database (see Appendix B), it is not clear if this divergence is significant, or an artefact of database bias.

Firm Survival Rates

The survival rate of new firms tends to decline exponentially over time (Fig. 13B). To replicate this behavior, we give our stochastic model an initial firm size distribution and then track firm survival over time. A firm 'dies' when it is reflected for the first time. At any given time, the firm survival rate is given by the fraction of firms that have never been reflected.

In order to model firm survival rates, we must choose an initial distribution of firms. We can make guesses about this distribution based on BLS *establishment* data. In 1994 — the first year the BLS tracked survival rates — the average size of new establishments was 7.3. In the same year, the average size of *all* US establishments was 16.9 (using data from

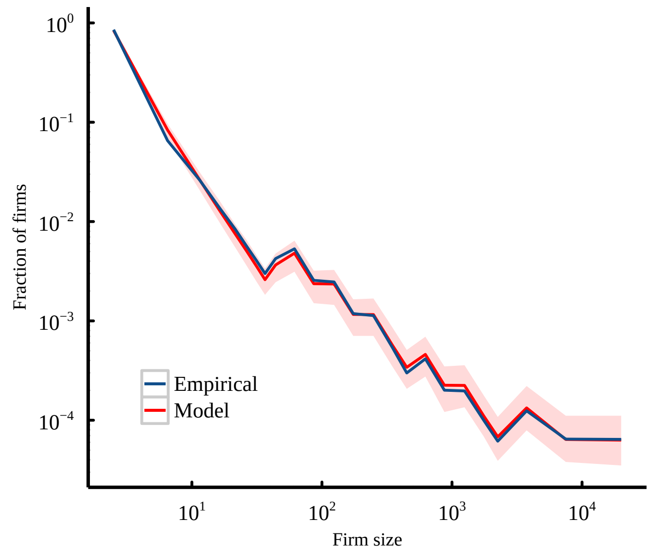

Figure 12: A Stochastic Model of US Firm Size Distribution

The US firm size distribution is shown for the year 2013 (blue line), along with a stochastic model (red) of 1 million firms with growth rates drawn from a truncated Laplace distribution with parameters $\mu = 0.99, \ \ b = 0.34$. The shaded region indicates the 90% confidence region of the model. US Data for employer firms is from the US Census Bureau, Statistics of U.S. Businesses (using data for 2013). This data is augmented with Bureau of Labor Statistics data for unincorporated self-employed workers (series LNU02032185 and LNU02032192). The histogram preserves Census firm-size bins, with self-employed data added to the first bin. The last point on the histogram consists of all firms with more than 10,000 employees. The model histogram uses Census bins to allow direct comparison.

Census Business Dynamics Statistics. It seems reasonable to assume that the average size of new *firms* might also be about half the average for all firms. It also seems reasonable to assume that the distribution of new firms can be modelled with a power law. Using these assumptions, I model the initial firm size distribution with a power law of $\alpha = 2.1$. This gives a mean size of close to 5 (about half the US average).

The empirical data shown in Figure 13 comes from the US Bureau of Labor Statistics (BLS). A caveat is that this data is for *establishment* (not *firm*) survival rates. An establishment refers to a specific business location, while a firm is a legal entity that may contain multiple establishments. For modelling purposes, I ignore this distinction here and assume that establishments are equivalent to firms.

Empirical and modelled survival rates are shown in Figure 13B). The survival rate model parameters ($\mu = 0.99, \ \ b = 0.35$) are nearly identical to the parameters ($\mu = 0.99, \ b = 0.34$) used to replicate the US firm size distribution (Fig. 12). These parameters are also consistent with the range estimated from Compustat data (Appendix D).

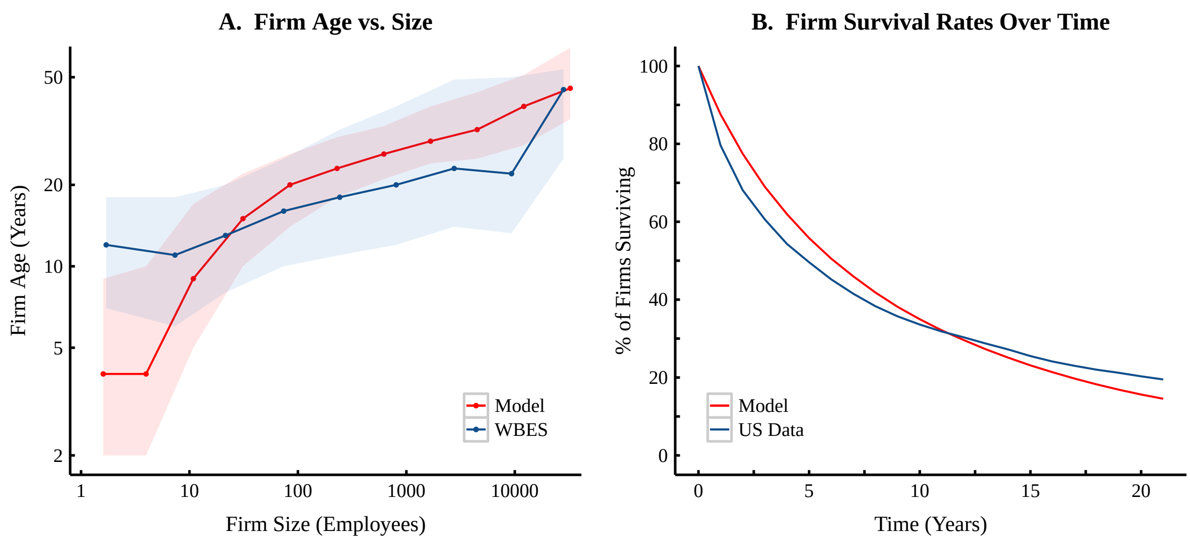

Figure 13: Stochastic Models Can Reproduce Firm Age/Survival Data

Panel A shows the relation between firm size and firm age within the World Bank Enterprise Survey (WBES) database (blue). A stochastic model (red) with growth rates drawn from a truncated Laplace distribution with parameters $\mu = 0.97, \;\; b = 0.55$ produces a similar firm size-age relation. Lines indicate medians and shaded regions indicate the interquartile range. Logarithmic bin locations are indicated with points. Panel B shows the survival rates of new firms over a period of 21 years. Empirical data (blue) is from the BLS Business Employment Dynamics database, Table 7, Survival of private sector establishments by opening year. The model (red) draws growth rates from a truncated Laplace distribution with parameters $\mu = 0.99, \;\; b = 0.35$.

G Bias and Error in the GDP Labor Time Method

My method for estimating power plant construction time is to take total cost and divide by (nominal) GDP per capita in the country and year of construction (henceforth called 'the GDP method'). In this section, I estimate the *bias* in this method. To do so, we need to investigate in detail the assumptions made by this approach.

The total cost of construction (C) of a power plant can be attributed to direct labor costs (L_d), profits and interest (denoted as K, for *capitalist* income), and non-labor costs (N):

$$C = L_d + K_d + N \tag{10}$$

By the rules of double-entry accounting, all non-labor costs will eventual become the *income* of other firms. Thus, after a long digression, we can eventually attribute non-labor costs to either indirect labor costs (L_i) or indirect capitalist costs (K_i):

$$C = L_d + L_i + K_d + K_i \tag{11}$$

Since we are not interested in differentiating between direct and indirect costs, we define L as the sum of direct and indirect labor costs, and K as the sum of direct and indirect capitalist costs:

$$C = L + K \tag{12}$$

Next, we define w as the average wage of all of the workers who are directly and indirectly involved in the construction project. Total labor cost (L) is then the average wage times total labor time (t). Substituting $L = w \cdot t$ into Eq. 12 gives:

$$C = w \cdot t + K \tag{13}$$

Solving for total labor time gives:

$$t = \frac{C - K}{w} \tag{14}$$

Equation 14 gives an accurate estimate of the total labor time involved in construction. Unfortunately, it is difficult (if not impossible) to calculate K (direct and indirect capitalist expenses) and w (the average wage of all direct and indirect workers). In order to get around this lack of data, I make the assumption that capitalist income can be neglected — that labor costs are approximately the same as total costs:

$$L = C - K \approx C \tag{15}$$

Furthermore, I assume that w is approximately the same as nominal GDP per capita (Y_{pc}).

$$w \approx Y_{pc} \tag{16}$$

Under these assumptions, Eq. 14 is approximated by Eq. 17:

$$t \approx \frac{C}{Y_{pc}} \tag{17}$$

By using GDP per capita as a measure of average income, we implicitly assume that all aspects of power plant construction occur within one country. For older plants, this is likely a good assumption. However, in the modern era of globalized production, this assumption is most likely violated to some degree, especially for key components of the plants like the generators and turbines. Unfortunately there is simply no way to disaggregate construction/manufacture costs to their various regions. However, we can correct for this bias to some degree by including power plants from as many nations as possible. The GDP method will then overestimate the labor time for plants constructed in developing countries (where GDP per capita is very low) and underestimate labor time for plants constructed in wealthy countries (where GDP per capita is very high). The hope is that these divergent biases will cancel themselves out.

How accurate is the GDP method? Unfortunately, we cannot compare GDP method estimates to the true labor time value (Eq. 14) because this latter formula contains *unknowable* quantities (K and w). However, we can test Eq. 14 against an alternative estimate for labor time that makes more accurate assumptions.

To proceed, let us first rewrite Eq. 14 as follows by factoring out C in the numerator:

$$t = \frac{C\left(1 - \dfrac{K}{C}\right)}{w} \tag{18}$$

We then make the assumption that capitalists involved (indirectly and directly) with the project earn profit and interest at approximately the *national average rate*. This means we assume that the capitalist share of total costs (K/C) is approximately the same as the capitalist share of national income (k_s).

$$\frac{K}{Y} \approx k_s \tag{19}$$

We also assume that workers involved (indirectly and directly) with the project earn the *national average wage* (w_n). Given these assumptions, Eq. 18 can be rewritten as:

$$t \approx \frac{C\,(1 - k_s)}{w_n} \tag{20}$$

We now have two way of estimating the labor time involved in the construction of a power plant (Eq. 17 and Eq. 20). Our expectation is that Eq. 20 is the more accurate estimate. To quantify the discrepancy between the two estimates, we construct an error ratio, which is the ratio of the two labor time estimates (Eq. 17 / Eq. 20):

$$\text{error ratio} = \frac{w_n}{Y_{pc}\,(1 - k_s)} \tag{21}$$

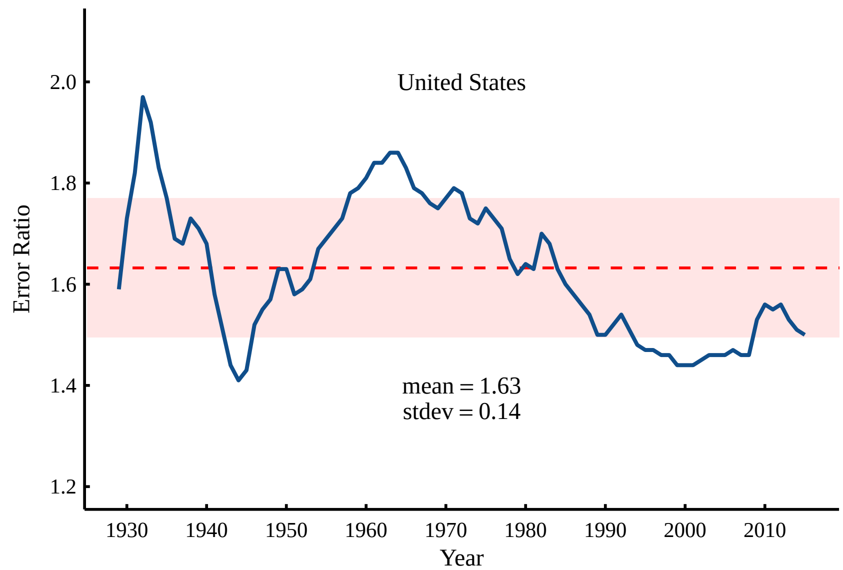

Figure 14: Error estimate of the GDP method for calculating labor time

This figure shows calculations of the error ratio (Eq. 21) using US data. All data is from the BEA. National income data is from Table 1.12, National Income by Type of Income. Capitalist share of national income is equal to profits (with CCA and IVA) and net interest divided by national income. The average wage is calculated by dividing the sum of the compensation of employees and proprietor income by the total persons engaged in production (Table 6.6B-D). US population data is from Maddison [1] and the World Bank series SP.POP.TOTL. Nominal GDP data is from the file *gdplev*.

Figure 14 shows this error ratio calculated using US data from 1929–2015. The results indicate that the GDP method (Eq. 17) *overestimates* labor time by roughly 60%. Why? By neglecting capitalist income, our estimate *inflates* the numerator in Eq. 14. Furthermore, GDP per capita is typically slightly lower than the average annual wage of a full-time worker, so the GDP method *deflates* the denominator in Eq. 14. Of course, this error estimate is itself based on the assumptions contained in Eq. 20. Still, it seems safe to conclude the following:

1. The GDP method likely *overestimates* the true labor time of power plant construction;
2. This overestimate is relatively stable over time.

Since our interest in this study is how labor time *scales* with plant capacity (and not with *absolute* labor time), this constant overestimate is of little concern. It will have no effect on the scaling of construction labor time with power plant size.

What is of more concern, however, are the *changes* in the error ratio that occur over time. How might this affect the estimation of power plant construction time? It is actually quite simple to model the effect of measurement error on a scaling relation. We begin by assuming that two variables, x and y, exhibit perfect power law scaling *identical* to that found between power plant capacity and construction labor time:

$$y = x^{1.26} \tag{22}$$

To study the effect of measurement error, we introduce a 'noise factor' ϵ (drawn from a lognormal distribution), that perturbs the perfect scaling relation:

$$y = x^{1.26} \cdot \epsilon \tag{23}$$

The effect of larger/smaller error can be modelled by increasing/decreasing the relative dispersion of ϵ. Suppose, for argument's sake, that Figure 14 *severely* underestimates the error associated with the GDP method. In reality, let us assume that the error is *10 times larger*. Since the relative standard deviation of the Figure 14 error ratio is 0.086, we can model the effect of a tenfold increase in error by setting ϵ to have a relative standard deviation of 0.86.

Figure 15 shows how the effects of this error factor (on our power law scaling relation) change as the orders of magnitude spanned by the dependent variable (x) increase. The horizontal axis shows the orders of magnitude spanned by the variable x, while the vertical axis shows the R^2 value of a log-log regression on the relation $y = x^{1.26} \cdot \epsilon$. The important result is that even though the measurement error is quite large, it becomes increasingly *inconsequential* as the data span increases.

Why? The R^2 value indicates the proportion of the variance in the dependent variable (y) that is predictable from the independent variable (x). Since we are conducting a log-log regression, it is helpful to look at the log transformed relation:

$$\log(y) = 1.26 \cdot \log(x) + \log(\epsilon) \tag{24}$$

Now, the variance in $\log(y)$ is affected both by the variance in $\log(x)$ and by the variance in $\log(\epsilon)$. But notice that the variance in both $\log(y)$ and $\log(x)$ will be proportional to the logarithm of the *range* of x. But this is equivalent to the orders of magnitude spanned by x (since orders of magnitude indicate scaling by factors of 10). Thus, the variance in $\log(x)$ and $\log(y)$ scales with the orders of magnitude spanned by x. However the variance in $\log(\epsilon)$ is *constant* — it does not change as the range of x increases. Because the variance in $\log(\epsilon)$ does not scale, its importance decreases as the range of x increases. That is, the fraction of variance in $\log(y)$ that is attributable to $\log(\epsilon)$ is inversely related to the orders of magnitude spanned by x.

So what does this result imply for the accuracy of the GDP method? Clearly, accuracy is a function of the orders or magnitude spanned by plant capacity. In our case study, plant capacity spanned *seven orders of magnitude*. According to Figure 15, even if the GDP method had a *severe* error factor (i.e. only accurate to within a factor of 3), the resulting measurement error would still not have a significant effect on the observed scaling relation. Thus, despite the error that is implicit in the GDP method, it is likely that our results are robust.

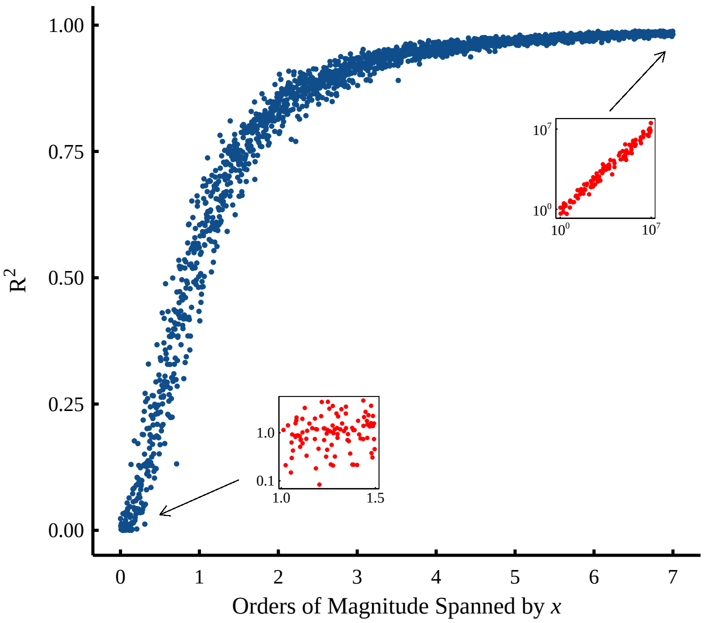

Figure 15: Data span vs. the effect of measurement error on a scaling relation

This figure shows multiple log-log regressions on data defined by the relation $y = x^{1.26} \cdot \epsilon$. Here x is a random variable whose logarithm is uniformly distributed, and ϵ is a noise factor drawn from a log-normal distribution with mean 1 and standard deviation 0.86 (which is 10 times the relative standard deviation of the error ratio in Fig 14). The horizontal axis shows the orders of magnitude spanned by the variable x, while the vertical axis shows the resulting R^2 value of the y vs. x regression. Each dot represents a *single* regression. Inset plots (red) show raw data underlying two different regressions — one with a small data span (bottom left) and one with a large data span (top right). For data that spans less than 2 orders of magnitude, the noise dominates the subsequent regression. However, once the span of x surpasses 4 orders of magnitude, the noise becomes inconsequential to the regression.

Still, given that the GDP method has a bias, why not use the more accurate approach given in Eq. 20? The problem with this formula is that it requires data on the capitalist share of national income as well as data on the average annual income of full time workers. This data is much more difficult to obtain than GDP per capita (especially in developing countries). Thus my use of the GDP method is mostly one of convenience: it makes analysis easier.

H A Hierarchical Model of the Firm

An 'ideal' hierarchy has a constant span of control throughout — meaning the employment ratio between each consecutive hierarchical level is *constant* (Fig. 16). This property allows total employment to be expressed as a geometric series of the span of control s. If the number of individuals in the top hierarchical level is a, and h_t is the total number of hierarchical levels, then total employment L is given by the following series:

$$L = a\left(1 + s + s^2 + ... + s^{h_t - 1}\right) \tag{25}$$

Using the formula for the sum of a geometric series, Eq. 25 can be rewritten as:

$$L = a\frac{1 - s^{h_t}}{1 - s} \tag{26}$$

We make the assumption that individuals in and above the hierarchical level h_m are considered *managers*. The number of managers M in a firm with h_t levels of hierarchy is equivalent to the employment of a firm with $h_t - h_m + 1$ levels of hierarchy:

$$M = a\frac{1 - s^{h_t - h_m + 1}}{1 - s} \tag{27}$$

We can use Eq. 27 and Eq. 26 to express management as a fraction of total employment (M/L):

$$\frac{M}{L} = \frac{1 - s^{h_t - h_m + 1}}{1 - s^h} \tag{28}$$

Asymptotic Behavior of the Management Fraction

The management fraction tends to grow with the number of hierarchical levels, but only to a certain point (Fig. 17). For $h_t > 10$ the management fraction approaches an asymptotic limit that depends only on the span of control s. Finding the asymptotic behavior of M/L requires evaluating the following limit:

$$\lim_{h_t \to \infty} \frac{M}{L} = \lim_{h_t \to \infty} \frac{1 - s^{h_t - h_m + 1}}{1 - s^{h_t}} \tag{29}$$

To evaluate this limit, I use L'Hospital's Rule, which states that $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$. We first rewrite Eq. 29 in a differentiable form, with a base e exponent:

$$\lim_{h_t \to \infty} \frac{M}{L} = \lim_{h_t \to \infty} \frac{1 - e^{\log(s)\cdot(h_t - h_m + 1)}}{1 - e^{\log(s)\cdot h_t}} \tag{30}$$

Applying L'Hospital's Rule, we take the derivative (with respect to h_t) of both the numerator and the denominator in Eq. 30, giving:

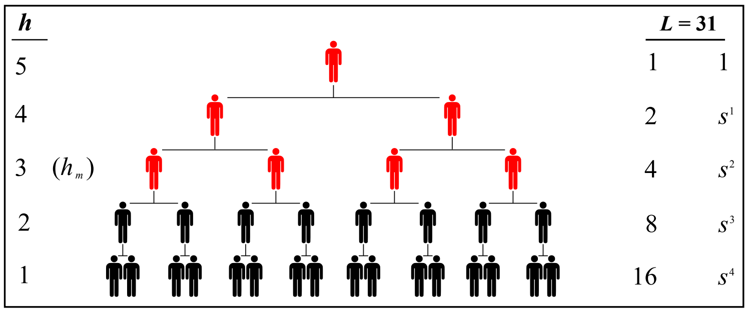

Figure 16: A Perfectly Hierarchical Firm

Within a perfectly hierarchical firm, the number of individuals in adjacent hierarchical levels differs by a factor of the span of control s (in this diagram, $s = 2$). This characteristic allows total employment L to be expressed as a geometric series of s. Managers (red) are defined as all individuals in and above level h_m (which equals 3 here).

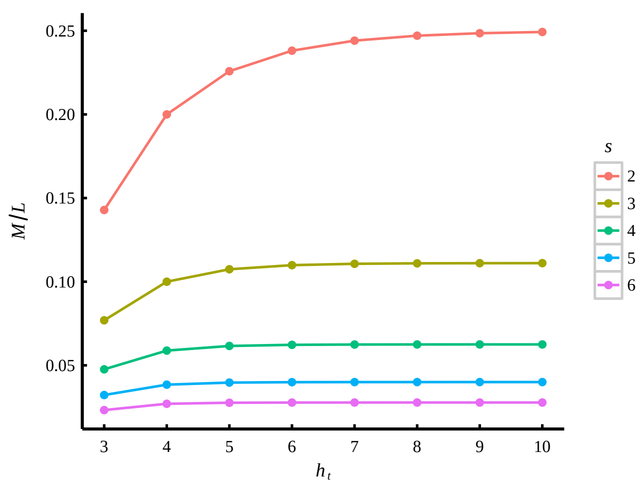

Figure 17: Asymptotic Behavior of the Management Fraction

This figure shows a plot of Eq. 28 for $h_m = 3$ and various s. As the total number of hierarchical levels (h_t) increases, the management fraction (M/L) within a firm grows rapidly, but soon reaches an asymptotic limit. This asymptote is a function of the span of control s, and the choice of h_m (the definition of where management begins).

$$\lim_{h_t \to \infty} \frac{M}{L} = \lim_{h_t \to \infty} \frac{-\log(s) \cdot e^{\log(s) \cdot (h_t - h_m + 1)}}{-\log(s) \cdot e^{\log(s) \cdot h_t}} \tag{31}$$

This simplifies to:

$$\lim_{h_t \to \infty} \frac{M}{L} \quad = e^{\log(s)(-h_m + 1)} \quad = s^{-h_m + 1} \tag{32}$$

Therefore, the asymptotic behavior of the management fraction depends only on the span of control, and our definition of management.

An Algorithm for Creating Hierarchies

The management model uses a power law simulated firm size distribution. In order to calculate the number of managers, each firm must be organized into hierarchical levels. I have developed the following algorithm to carry out this process.

Having selected a firm, we know its employment L and its span of control s; however, the total number of hierarchical levels h_t is unknown. To calculate h_t, we assume, for the moment, that the size of the top hierarchical level is one. Therefore, h_t must satisfy:

$$L = \frac{1 - s^{h_t}}{1 - s} \tag{33}$$

Solving for h_t gives:

$$h_t = \frac{\log\left[1 + L(s-1)\right]}{\log(s)} \tag{34}$$

Since h_t must be discrete, we round the solution to the nearest integer. My method is then to 'build' the hierarchy from the bottom up. If the bottom hierarchical level contains b workers, then L is defined by the series:

$$L = b\left(1 + \frac{1}{s} + \frac{1}{s^2} + ... + \frac{1}{s^{h_t - 1}}\right) \tag{35}$$

Using the formula for the sum of a geometric series, this becomes:

$$L = b\,\frac{1 - 1/s^{h_t}}{1 - 1/s} \tag{36}$$

At the moment, L is known but b is unknown. We therefore solve for b (and round the answer to the nearest integer):

$$b = L\,\frac{1 - 1/s}{1 - 1/s^{h_t}} \tag{37}$$

Once we have b, we can differentiate the firm into hierarchical levels by dividing b by powers of s (Eq. 35). Due to rounding errors, the sum of the employment of all hierarchical levels may differ from the original firm size L. Any discrepancies are added (or subtracted) to the base level to give the correct firm size. The number of managers M is then simply the sum from hierarchical level h_m to h_t.

I An Agrarian Model of Institution Size

In this section, I use an adaptation of the hierarchical firm model (used in Fig. 7 of the main paper and discussed in Appendix H) to explain the institution size limits posed by an agrarian economy. In agrarian societies, the *vast* majority of the population is directly engaged in agricultural activities — a direct result of low agricultural labor productivity. This model aims to demonstrate that the large size of the agricultural population places inherent constraints on agrarian institution size. The model makes the following assumptions:

1. All agrarian institutions are 'ideal' hierarchies with the same span of control.
2. The agricultural population forms the bottom hierarchical level of *all* institutions.
3. Agrarian institution sizes are distributed according to a power law.

The model is depicted graphically in Figure 18. In formulating this model, I have in mind a feudal society in which the institutional unit can be loosely thought of as the feudal manor. These institutions are organized around the extraction of an agricultural surplus from peasants/serfs, and are defined by a rigid caste system (with serfs at the bottom). For the sake of simplicity, we assume that all peasants/serfs are engaged in agriculture.

There is evidence that feudal manors (like modern firms) were power-law distributed. For instance Hegyi et al. find an approximate power law distribution of serf ownership by nobles/aristocrats in 16th century Hungary [36]. Similarly, Kahan finds a highly skewed distribution of serf ownership in 18th century Russia [37] (although this distribution is better fit with a lognormal function).

Although the above assumptions may well be wrong (or oversimplifications), this model is intended mostly as a thought experiment. Figure 19A shows the modelled relation between the agricultural portion of the total population and mean institution size (with the span of control varying between 2 and 3). The prediction is that the agricultural population should decline rapidly as mean institution size increases.

In this model, the fraction of the population engaged in agriculture places strict limitations on institution size. Estimates vary on the size of this agricultural fraction of the population in historical agrarian societies. In Figure 19, I use Cottrell's estimate that 95% of the population in ancient Egypt was directly engaged in agricultural activity [38] (indicated by the red horizontal line in Fig. 19A). According to the model, this limits mean institution size to between 1.2 and 1.32 people (indicated by the grey region).

If we further assume that the modern relation between mean firm size and energy use per capita is applicable to agrarian institutions, we can make predictions about rates of energy consumption. We input the estimated mean institution size range into the firm size versus energy regression from Figure 1C (main paper) to predict a range of energy use per capita for this model society (Fig. 19B).

The predicted interval of roughly 10 to 30 GJ per capita is a surprisingly realistic range for a typical agrarian society. For instance Warde estimates that England used 20 GJ of energy per capita in 1560 [39]. Similarly, Malanima estimates that 1st and 2nd century Romans consumed between 9 and 17 GJ per capita [40].

Figure 18: The Decline of Agricultural Workers as a Function of Institution Size in an Agrarian Economy

This figure shows an adaptation of the hierarchical firm model presented in Appendix H. In agrarian societies, we assume that the bottom hierarchical level of all institutions is constituted entirely of agricultural workers. As institution size increases, the relative size of the agrarian population declines. All institutions are assumed to be 'ideal' hierarchies with constant spans of control. In the model (not accurately represented here) the institution size is distributed according to a power law.

This model can be used to understand how energetic constraints place limits on institution size within agrarian societies. In all societies, the relative size of the agricultural population is a function of agricultural labor *productivity* [41]. The agriculture sector must produce a surplus of food in order to feed the non-agricultural population [42]. It follows that the fraction of workers in agriculture can decline only if their per person output of surplus food increases.

In agrarian societies, agricultural workers relied exclusively on human and animal labor, which meant that output per worker was extremely low compared to modern industrial agriculture. The result was that the agricultural surplus was very small, allowing only a small non-agricultural population to exist [43]. According to our model, this leads to inherent constraint on institution size.

Agricultural productivity, in turn, is directly related to energy use. Increasing agricultural labour productivity requires that each worker convert more energy into useful work. Historically, this meant first introducing more draft animals per worker, followed by the widespread adoption of fossil fuel powered equipment (tractors, combines, etc.). As agricultural workers increase their energy use, this will impact per capita energy use for society at large.

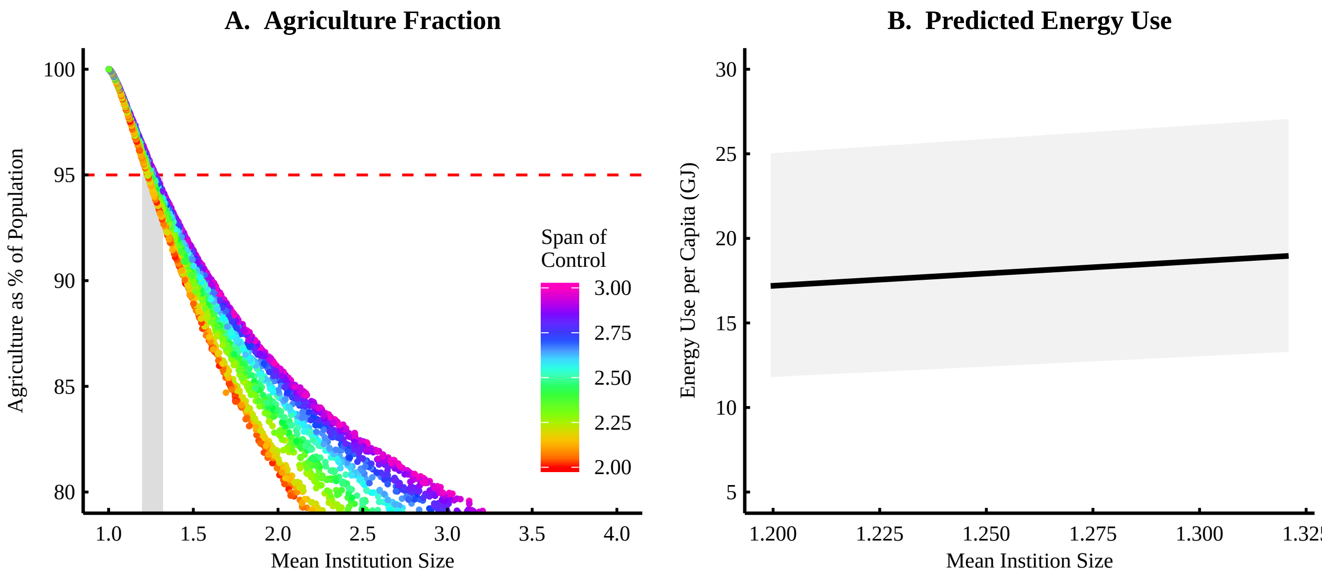

Figure 19: Modelling Agricultural Constraints on Institution Size, and the Implication for Energy Use per Capita in Agrarian Societies

This figure shows how a hierarchical model of an agrarian society can be used to relate the size of the agricultural population to institution size and energy use per capita. Panel A shows the modelled relation between the agricultural portion of the population and mean institution size. Different mean institution sizes are generated by varying the exponent of the institution size distribution. Different spans of control are indicated by color. The red horizontal line corresponds to a society with 95% of the population in agriculture, and the shaded region shows the corresponding prediction for mean institution size. Panel B shows the energy use per capita predictions for this range of institution size. These predictions are made using the national mean firm size vs. energy use per capita regression shown in Fig. 1C of the main paper. The formula is $E_{pc} = 14.3\bar{L}^{1.02}$, where E_{pc} is energy per capita and $\bar{L}$ is mean firm size. The grey region indicates the 95% confidence interval of the prediction.

Unfortunately, this model cannot be used to study the transformation from an agrarian to an industrial society because its premise *breaks down* as this transition proceeds. The model is based on a feudal society organized around the expropriation of an agricultural surplus from a serf/peasant class. As feudal relations give way to market relations, this social structure ceases to exist. New institutions form that have nothing to do with agriculture, meaning assumption 2 (the bottom level of all institutions is entirely made up of agricultural workers) becomes *absurd*.

Despite its shortcomings, this model is useful for understanding the possible limitations placed on institution size by the energetic constraints of an agrarian economy.

References

[1] Maddison A. Statistics on World Population, GDP and Per Capita GDP, 1-2008 AD; 2008. Available from: http://www.ggdc.net/maddison/Maddison.htm.

[2] Efron B, Tibshirani RJ. An introduction to the bootstrap. London: CRC press; 1994.

[3] Carter SB, Gartner SS, Haines MR, Olmstead AL, Sutch R, Wright G. Historical Statistics of the United States: Millennial Edition. Cambridge: Cambridge University Press; 2006.

[4] Nitzan J, Bichler S. Capital as Power: A Study of Order and Creorder. New York: Routledge; 2009.

[5] Ariga K, Brunello G, Ohkusa Y, Nishiyama Y. Corporate hierarchy, promotion, and firm growth: Japanese internal labor market in transition. Journal of the Japanese and International Economies. 1992;6(4):440–471.

[6] Audas R, Barmby T, Treble J. Luck, effort, and reward in an organizational hierarchy. Journal of Labor Economics. 2004;22(2):379–395.

[7] Baker G, Gibbs M, Holmstrom B. Hierarchies and compensation: A case study. European Economic Review. 1993;37(2-3):366–378.

[8] Bell B, Van Reenen J. Firm performance and wages: evidence from across the corporate hierarchy. CEP Discussion paper. 2012;1088.

[9] Dohmen TJ, Kriechel B, Pfann GA. Monkey bars and ladders: The importance of lateral and vertical job mobility in internal labor market careers. Journal of Population Economics. 2004;17(2):193–228.

[10] Eriksson T. Executive compensation and tournament theory: Empirical tests on Danish data. Journal of Labor Economics. 1999;17(2):262–280.

[11] Heyman F. Pay inequality and firm performance: evidence from matched employeremployee data. Applied Economics. 2005;37(11):1313–1327.

[12] Lima F. Internal Labor Markets: A Case Study. FEUNL Working Paper. 2000;378.

[13] Morais F, Kakabadse NK. The Corporate Gini Index (CGI) determinants and advantages: Lessons from a multinational retail company case study. International Journal of Disclosure and Governance. 2014;11(4):380–397.

[14] Mueller HM, Ouimet PP, Simintzi E. Within-Firm Pay Inequality. SSRN Working Paper. 2016;.

[15] Rajan RG, Wulf J. The flattening firm: Evidence from panel data on the changing nature of corporate hierarchies. The Review of Economics and Statistics. 2006;88(4):759–773.

[16] Treble J, Van Gameren E, Bridges S, Barmby T. The internal economics of the firm: further evidence from personnel data. Labour Economics. 2001;8(5):531–552.

[17] Hipple S. Self-employment in the United States: an update. Monthly Lab Review. 2004;127:13.

[18] Sandefur J. On the evolution of the firm size distribution in an African economy. Center for the Study of African Economies Working Paper Series. 2010;5.

[19] Hasan R, Jandoc KR. The Distribution of Firm Size in India: What Can Survey Data Tell Us? Asian Development Bank Economics Working Paper Series. 2010;(213).

[20] Angelini P, Generale A. On the evolution of firm size distributions. The American Economic Review. 2008;98(1):426–438.

[21] Cabral LM, Mata J. On the evolution of the firm size distribution: Facts and theory. American economic review. 2003;93(4):1075–1090.

[22] Gibrat R. Les inegalites economiques. Recueil Sirey; 1931.

[23] Gupta HM, Campanha JR, de Aguiar DR, Queiroz GA, Raheja CG. Gradually truncated log-normal in USA publicly traded firm size distribution. Physica A: Statistical Mechanics and its Applications. 2007;375(2):643–650.

[24] Hart PE, Prais SJ. The analysis of business concentration: a statistical approach. Journal of the Royal Statistical Society Series A (General). 1956;119(2):150–191.

[25] Stanley MH, Buldyrev SV, Havlin S, Mantegna RN, Salinger MA, Stanley HE. Zipf plots and the size distribution of firms. Economics letters. 1995;49(4):453–457.

[26] Axtell RL. Zipf distribution of US firm sizes. Science. 2001;293:1818–1820.

[27] Gaffeo E, Gallegati M, Palestrini A. On the size distribution of firms: additional evidence from the G7 countries. Physica A: Statistical Mechanics and its Applications. 2003;324(12):117–123. doi:10.1016/S0378-4371(02)01890-3.

[28] Clauset A, Shalizi CR, Newman ME. Power-law distributions in empirical data. SIAM review. 2009;51(4):661–703.

[29] Stanley MH, Amaral LA, Buldyrev SV, Havlin S, Leschhorn H, Maass P, et al. Scaling behaviour in the growth of companies. Nature. 1996;379(6568):804–806.

[30] Kaldor N. A model of economic growth. The Economic Journal. 1957;67(268):591–624.

[31] Feynman RP, Leighton RB, Sands M. The Feynman Lectures on Physics, Desktop Edition Volume I. vol. 1. Basic books; 2013.

[32] Kesten H. Random difference equations and Renewal theory for products of random matrices. Acta Mathematica. 1973;131(1):207–248. doi:10.1007/BF02392040.

[33] Biham O, Malcai O, Levy M, Solomon S. Generic emergence of power law distributions and Levy-Stable intermittent fluctuations in discrete logistic systems. Physical Review E. 1998;58(2):1352.

[34] Gabaix X. Zipf's Law for Cities: An Explanation. The Quarterly Journal of Economics. 1999;114(3):739–767. doi:10.1162/003355399556133.

[35] Harrison JM. Brownian motion and stochastic flow systems. New York: Wiley; 1985.

[36] Hegyi G, Neda Z, Santos MA. Wealth distribution and Pareto's law in the Hungarian medieval society. Physica A: Statistical Mechanics and its Applications. 2007;380(1):271–277.

[37] Kahan A, Hellie R. The plow, the hammer, and the knout: An economic history of eighteenth-century Russia. Chicago: University of Chicago Press; 1985.

[38] Cottrell F. Energy & Society (Revised): The Relation Between Energy, Social Change, and Economic Development. Bloomington: AuthorHouse; 2009.

[39] Warde P. Energy Consumption in England & Wales, 1560-2000. Consiglio nazionale delle ricerche, Istituto di studi sulle societa del Mediterraneo; 2007.

[40] Malanima P. Energy Consumption in the Roman World. In: The Ancient Mediterranean Environment between Science and History. Boston: Brill; 2013. p. 13–36.

[41] Fix B. Rethinking Economic Growth Theory from a Biophysical Perspective. Hall C, editor. New York: Springer; 2015.

[42] Giampietro M, Mayumi K, Sorman A. The Metabolic Pattern of Societies: Where Economists Fall Short. New York: Routledge; 2012.

[43] Pimentel D, Pimentel MH. Food, Energy, and Society, Third Edition. New York: CRC Press; 2007.

[44] Cleveland C, Costanza R, Hall C, Kaufmann R. Energy and the US economy: a biophysical perspective. Science. 1984;225(4665):890–897.