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Hierarchical Power as the Basis of Income and Class

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How the Rich Are Different:

Hierarchical Power as the Basis of Income and Class

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Abstract

What makes the rich different? Are they more productive, as mainstream economists claim? I offer another explanation. What makes the rich different, I propose, is *hierarchical power*. The rich command hierarchies. The poor do not. It is this greater control over subordinates, I hypothesize, that explains the income and class of the very rich. I test this idea using evidence from US CEOs. I find that the relative income of CEOs increases with their hierarchical power, as does the capitalist portion of their income. This suggests that among CEOs, both income *size* and income *class* relate to hierarchical power. I then use a numerical model to test if the CEO evidence extends to the US general public. The model suggest that this is plausible. Using this model, I infer the relation between income size, income class, and hierarchical power among the US public. The results suggests that behind the income and class of the very rich lies immense hierarchical power.

JEL Subject Codes: D31 (Personal Income, Wealth, and Their Distributions); D33 (Factor Income Distribution); B5 (Current Heterodox Approaches);

Keywords: hierarchy; power; functional income distribution; personal income distribution; inequality; capital as power; class

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1 Introduction

Let me tell you about the very rich. They are different from you and me.

— F. Scott Fitzgerald (1926)

Yes, they have more money.

— Ernest Hemingway (1936)

What makes the rich *different*? Mainstream economists think the rich are more *productive*. I offer another explanation. What makes the rich different, I propose, is *hierarchical power*. The rich command hierarchies. The poor do not. This difference, I believe, is key to understanding both *income* and *class*. The purpose of this paper is to test this idea.

My thinking is based on a simple principle. I propose that individuals tend to use their *power* — their influence over others — to access resources. Of course many political economists have had this idea before. But it has always been difficult to test. The problem is that power is complex and multifaceted, which means it often defies measurement. And without quantification, it is difficult to show that power affects income (access to resources).

I argue that focusing on *hierarchical* power solves (at least in part) this measurement problem. Within a hierarchy, the chain of command defines who has control over whom. We can use this control over subordinates to define a simple measure of hierarchical power. This allows us to test how hierarchical power relates to income.

And we can do more. I propose that hierarchical power also relates to *class*. To understand this thinking, we must question some entrenched ideas. Political economists usually define class in terms of the ownership of *things* — what Marxists call the 'means of production'. *Capitalists* own the means of production. *Laborers* do not. In contrast, Nitzan and Bichler (2009) treat ownership as an *act of power*. This means they focus on the ownership of *institutions*. Drawing on Nitzan and Bicher's work, I argue that ownership is an ideology that legitimizes hierarchy. Thus *income* from ownership (capitalist income) should relate to hierarchical power.

Using these ideas, I create two joint hypotheses (Section 2). The first links hierarchical power to income *size*:

> **Power-Income Hypothesis**: Individual income is proportional to hierarchical power.

The second hypothesis links hierarchical power to income *class* — the class-based composition of individual income:

> **Capitalist Gradient Hypothesis**: The portion of individuals' income that comes from ownership is proportional to hierarchical power.

Together, these hypotheses use hierarchical power to unite the study of income distribution. The question is, are they true?

After outlining methods (Section 3), I test these hypotheses in two ways. I first conduct a case study of American CEOs (Section 4). I focus on CEOs because their position as corporate commanders allows a shortcut for measuring hierarchical power. We can estimate the hierarchical power of CEOs from firm size alone. Using this method, I find that CEOs' income increases with hierarchical power, as does the capitalist *portion* of their income. Thus the CEO evidence supports both the power-income and capitalist gradient hypotheses.

This result is promising, but also unsatisfying. We want to know if the relation between CEOs' income size, income type, and hierarchical power is *also* found in the general public. Unfortunately, we lack the data to test this directly. To get around this data shortage, I use a model to make inferences (Section 5). The model predicts the distribution of income that should occur if CEO trends (i.e. the relation between income size, income type, and hierarchical power) are *also* found among the US public. If the predicted income distribution is consistent with US data, we infer that our results plausibly extend beyond CEOs. I find that the model predicts with reasonable accuracy the distribution of US *labor* income, *capitalist* income, and total *personal* income. The model also predicts the 'hockey-stick' relation between income *size* and income *class* (the capitalist component of individual income). Among the top 1% or earners, the capitalist fraction of income explodes.

Since its predictions are reasonably consistent with US data, I use the model to make the first inference about how US income and class relate to hierarchical power (Section 6). The results underscore Fitzgerald's famous words that the *rich are different*. Or as Tim Di Muzio writes, there is the '1% and the rest of us' (2015). Not only do the 1% earn more, they are far more likely to be *capitalists*. This we know from the seminal work of Thomas Piketty (2014). But what we have lacked is a reason *why* income and class are related. The model suggests a simple explanation. Behind the income and class of the very rich, I infer, lies *immense hierarchical power*. This incendiary result suggests that the root of the inequality problem may be the hierarchical power of the rich.

1.1 The Problem

This paper is motivated by a simple problem. We have no theory that adequately explains both income *size* (how much one earns) and income *class* (the source of one's income). We have three types of income theory (below). Each have problems.

1.	Core Theories:	Marxist and neoclassical political economy
2.	Stochastic Models:	Models that generate skewed distributions using random shocks to individual income
3.	Power Theories:	Mostly qualitative descriptions of how power affects income

The hallmark of our core theories — Marxist and neoclassical political economy — is that they both assume *value is produced*. Neoclassical economists think both laborers and capitalists produce value. Each 'factor of production' then earns its (marginal) contribution to output (Clark, 1899; Wicksteed, 1894). Marxists agree that laborers produce value, but have different ideas about capitalists. According to Marx (1867), capitalists earn income by *exploiting* workers.

There are many problems with our core theories that I will not review here.[1] Instead, my concern is what they conclude about *personal* income (i.e. the size distribution of income). Both neoclassical and Marxist theories agree that labor produces value, and that this is the source of labor income. Using this reasoning, neoclassical economists (Becker, 1962; Mincer, 1958; Schultz, 1961) and Marxists (Rubin, 1973) have concluded the same thing: if two workers earn different *incomes*, they must have different *productivity*.[2] If we generalize this reasoning, it implies that workers' productivity should be as unequally distributed as their income. Yet this is not true. When workers' productivity is measured objectively, it fails to explain differences in income (Fix, 2018e). This leaves the labor productivity aspect of neoclassical and Marxist theories at odds with the evidence.

[1] For problems with marginal productivity theory, see Cohen and Harcourt (2003); Felipe and Fisher (2003); Harcourt (2015); Hodgson (2005); Nitzan and Bichler (2009); Pullen (2009); Robinson (1953); Sraffa (1960). For problems with Marxist theory, see Nitzan and Bichler (2009); Samuelson (1971).

[2] There is a subtle distinction between neoclassical and Marxist theory. Neoclassical theory attributes labor income *directly* to productivity. But Marxist theory attributes income to the *value* of labor power. The latter is the labor time required to reproduce labor power. Since the labor power of more productive workers tends to take more to reproduce, more productive workers tend to earn higher wages.

In hindsight, this is understandable. The facts of personal income were discovered *after* our core theories of income distribution were developed. It was late in the 19th century when Vilfredo Pareto (1897) showed that personal income was skewed and followed a power-law distribution. When these facts became well known (in the 20th century), neoclassical and Marxist theories of income were already firmly in place.

While political economists were slow to react to Pareto's discovery, mathematicians quickly looked for processes that could generate skewed distributions. They soon found that a simple random process could do the trick. These 'stochastic models' assume that an individual's income changes randomly over time. When we apply this process to many individuals, it creates a skewed distribution of income. For early models, see Champernowne (1953), Simon (1955), and Rutherford (1955). For more recent work, see Gabaix et al. (2016), Nirei and Aoki (2016), and Toda (2012).

These models are important because they show how the dynamics of individual income can lead to inequality. Yet they are scientifically unsatisfying. They do not explain the income of *specific* individuals — something we want an income distribution theory to do. And since stochastic models deal only with isolated individuals (rather than groups of individuals), they are not helpful for understanding how income relates to class.

That brings us to income theories based on *power*. These are the most promising theories (in my opinion), but also the most marginalized and underdeveloped. An incomplete list of people who have linked income to power would include Berle and Means (1932), Brown (1988), Commons (1924), Dugger (1989), Galbraith (1985), Huber et al. (2017), Lenski (1966), Mills (1956), Munkirs (1985), Nitzan and Bichler (2009), Peach (1987), Sidanius and Pratto (2001), Tool and Samuels (1989), Tool (2017), Veblen (1904, 1923), Weber (1978), and Wright (1979).

Power theories differ wildly in their development and scope. But the general idea is that income inequality is caused by the concentration of power. I find this hypothesis compelling. It avoids the trap of attributing income to productivity. And it avoids the atomism of stochastic models. But it still has a problem. How do we *measure* power? And how do we measure it in a way that relates to both *individuals* and *classes*? This has been a major sticking point. Power surrounds us, but it often defies measurement.

The solution, I argue, is to reduce our scope. To measure power, we focus only on *hierarchical power* — the control over subordinates within a hierarchy. We then use this limited concept of power to create a joint theory of income *size* and income *class*.

2 Hierarchical Power as the Basis for Income and Class

I propose that hierarchical power can explain both income *size* and income *class*. I focus on hierarchical power for the following reasons:

1. Hierarchy is *ubiquitous* (Section 2.1)
2. Hierarchical power can be *measured* (Section 2.2)
3. Hierarchical power plausibly relates to income *size* (Section 2.3)
4. Hierarchical power plausibly relates to income *class* (Section 2.4)

2.1 Hierarchy's Ubiquity

Price and Feinman (1995; 2010) argue that for the last 5000 years, hierarchy has been the dominant way of organizing society. True, some traditional societies lack *formal* hierarchy. But these societies still have *informal* ranking, if only by sex or age (Ames, 2010).

The urge to form hierarchies likely has evolutionary origins. Virtually all social mammals form dominance hierarchies (Barroso et al., 2000; Guhl et al., 1945; Kondo and Hurnik, 1990; Meese and Ewbank, 1973; Sapolsky, 2005; Uhrich, 1938). In these hierarchies, high social status allows greater access to resources, particularly sexual mates (Bradley et al., 2005; Cowlishaw and Dunbar, 1991; Haley et al., 1994; Girman et al., 1997; Gerloff et al., 1999; Wroblewski et al., 2009). Given our evolutionary heritage, we expect to find similar behavior in humans.

Unsurprisingly, studies show that when children are placed in small groups, they quickly form hierarchies (Frankel and Arbel, 1980; Savin-Williams, 1980; Strayer and Trudel, 1984). And like animals, humans with higher status tend to have more children (Betzig, 1982, 2012, 2018; Cronk, 1991; Mealey, 1985). This hints that like other animals, humans use hierarchy to distribute resources.

2.2 Measuring Hierarchical Power

In this paper, I focus on a single dimension of power — the control over subordinates within a hierarchy. I call this *hierarchical power*. In a hierarchy, control over subordinates is dictated by the chain of command. Superiors have power over their *direct* subordinates, but also their *indirect* subordinates (their subordinates' subordinates). According to this definition, the more subordinates one

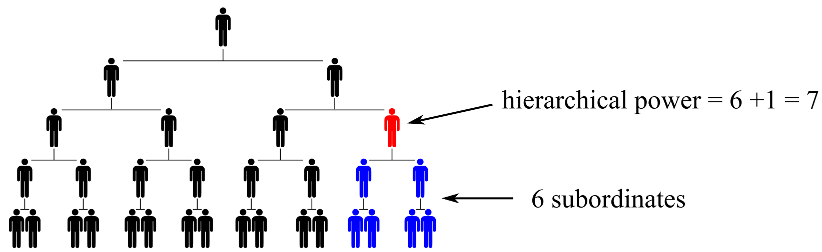

Figure 1: Measuring Hierarchical Power

This figure illustrates the calculation of hierarchical power. The red individual has 6 subordinates (blue). Using Eq. 1, the hierarchical power of this person equals 7.

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has, the greater one's hierarchical power. I measure hierarchical power as proportional to the total number of subordinates under one's control:

$$\text{hierarchical power} = \text{total number of subordinates} + 1 \tag{1}$$

All individuals start with a power of 1, indicating they have control over themselves. Hierarchical power is then proportional to the total number of subordinates under one's control. Figure 1 shows an example calculation.

This definition of hierarchical power is *structural* — it deals only with one's abstract position within a hierarchy. Many other qualities might affect power, such as a leader's charisma, subordinates' trust in superiors, ideologies, laws, and so on. While important, these qualities are difficult to measure so I exclude them. My definition of hierarchical power is also *downward* looking only. It looks only at subordinates — it does not care if an individual has superiors. This means we treat a mid-level manager in a large firm the same as a CEO in a small firm (if they both have the same number of subordinates). We ignore the fact that the mid-level manager *takes* orders but the CEO does not.

2.3 Hierarchical Power and Income

I propose that hierarchical power strongly affects income. The idea is that individuals use their power within a hierarchy to access resources. I call this the power-income hypothesis:

> **Power-Income Hypothesis**: Individual income is proportional to hierarchical power (as measured by Eq. 1).

The power-income hypothesis may seem no different than an education-income hypothesis or a race-income hypothesis (and so on). These hypotheses

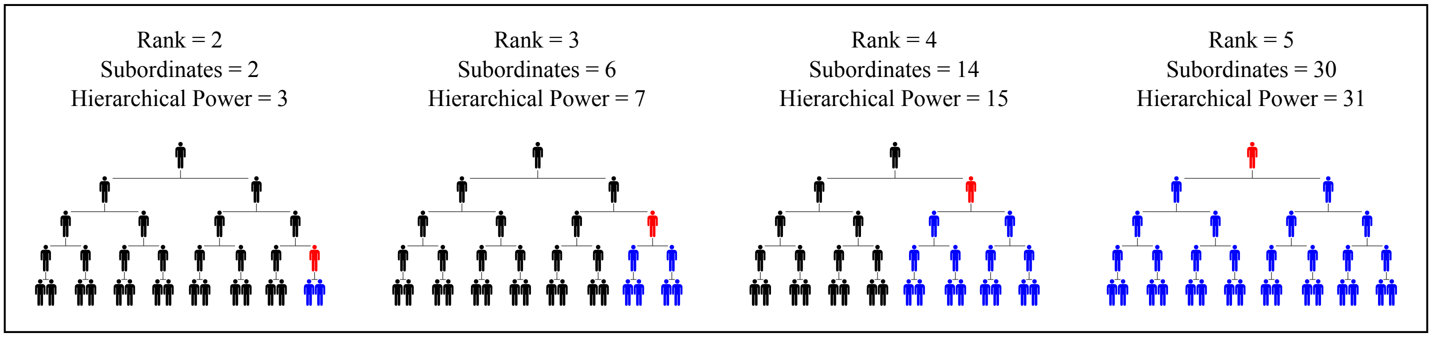

Figure 2: The Exponential Growth of Hierarchical Power with Rank

In an idealized hierarchy, the total number of subordinates (blue) grows *exponentially* with hierarchical rank (red). The exact relation depends on the 'span of control' — the number of subordinates directly below each superior.

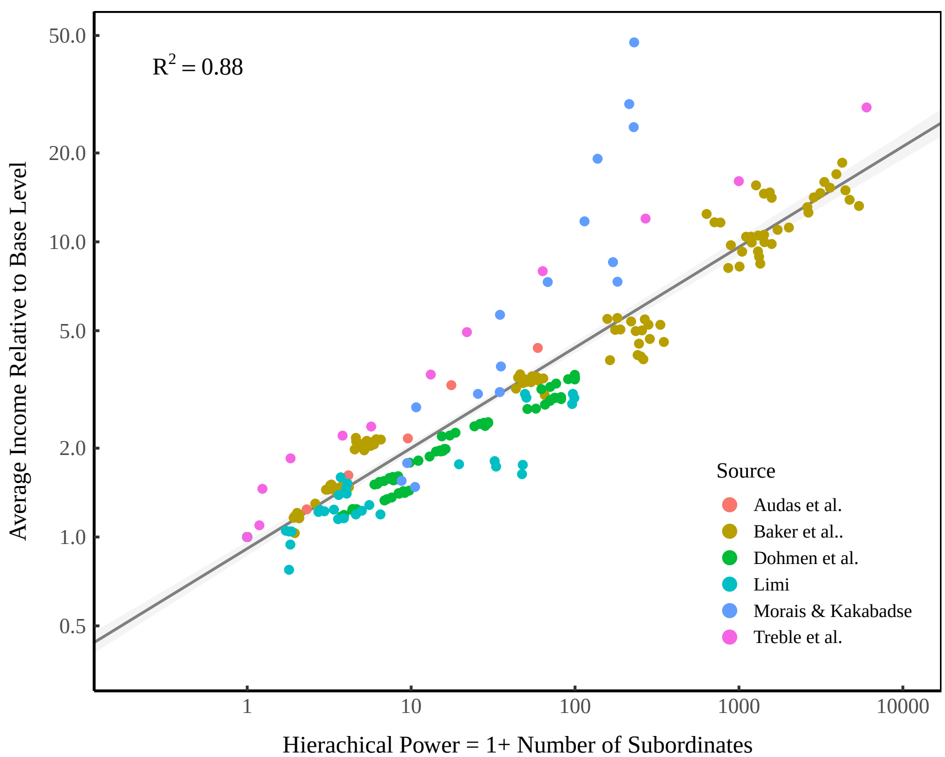

Figure 3: Average Income vs. Hierarchical Power Within Case-Study Firms

This figure shows data from six firm case studies (Audas et al., 2004; Baker et al., 1993; Dohmen et al., 2004; Lima, 2000; Morais and Kakabadse, 2014; Treble et al., 2001). The vertical axis shows average income within each hierarchical level of the firm. Incomes are normalized so the base-level income equals one. The horizontal axis shows average hierarchical power of the individuals in each rank. This is equal to one plus the average number of subordinates below a given hierarchical level. Each point represents a single firm-year observation, and color indicates the particular case study. Grey regions around the regression indicate the 95% prediction interval. See the Appendix and Fix (2018d) for a detailed discussion of sources and methods.

all identify something that plausibly affects income. But beneath the surface, there is a big difference. Hierarchical power is a *social* characteristic. You have hierarchical power because *others* follow your orders. Education and race, in contrast, are *individual* characteristics. They belong exclusively to the individual. Why does this matter? Because individual characteristics are constrained by variation between individuals, which are usually *small*. Social characteristics are not. This difference is important.

Let's use an example to compare the *individual* characteristic of education to the *social* characteristic of hierarchical power. Steve Easterbrook, the CEO of McDonald's, has a Bachelor's degree. While he likely has more education than the average McDonald's employee, the difference is small. If the average employee has a high-school diploma, Easterbrook would have about 30% more years of formal education. Now consider Easterbrook's hierarchical power. As CEO, Easterbrook commands the entire McDonald's workforce. In 2016, this meant he had 375,000 subordinates. In contrast, the average McDonald's employee likely has *none*. Thus Easterbrook likely has *hundreds of thousands* of times more hierarchical power.

This is the crucial part of the power-income hypothesis. Hierarchical power can vary immensely between individuals. This is because it grows *exponentially* with rank (Fig. 2). If income is proportional to hierarchical power, this should lead to vast inequalities. Returning to our example, in 2016 Steve Easterbrook earned over 600 times the pay of an average McDonald's employee. His enormous hierarchical power can plausibly explain this disparity. His modestly better education cannot.[3]

This is the *reasoning* behind the power-income hypothesis. But its validity is an *empirical* question that I test in Sections 4 and 5. Here I review the existing evidence. Fix (2018d) shows that income within six case-study firms is strongly related to hierarchical power (Fig. 3). Does this relation extend to other firms, and to the general public? I attempt to answer this question in Sections 4 and 5.

2.4 Hierarchical Power and Class

As well as explain income *size*, I think hierarchical power can explain income *class*. To make this connection, I reflect on the ideology of power. My thinking is heavily influenced by the work of Nitzan and Bichler (2009) and their concept of 'capital as power'.

[3] Data for Easterbrook's pay is from Execucomp. Data for the number of McDonald's employees and average employee pay are from Compustat.

Nitzan and Bichler observe that all societies have ideologies that legitimize power. Traditional societies justify power through *kinship* — tracing lineage to a common ancestor (Kirchhoff, 1955; Sahlins, 1963). Feudal societies use *religion* (Hunt, 2016). Capitalist societies, according to Nitzan and Bichler, use *ownership* to justify power.

In each system, the ideology does three things. First, it legitimizes the power of rulers. Second, it justifies their income. Third, it often creates a distinct income *class* for the rulers. For instance, a feudal king's power and income are justified by God (the divine right of kings). And as ruler, the king's income is given its own class — *taxes*.

When moving from the feudalism to capitalism, Nitzan and Bichler argue that there is one big difference. In capitalism, the ideology of ownership legitimizes the *buying* and *selling* of power. But the other attributes of ideology remain the same. The ideology of ownership continues to justify the power and income of rulers, and gives their income a separate class. Owners earn *profit* — a class of income that borders on sacred in capitalism. Non-owners earn *wages*. Using this reasoning, Nitzan and Bichler argue that income from ownership relates to power. I take this reasoning and apply it to hierarchies.

Capitalist Income and Hierarchical Power

Generations of political economists have focused on the ownership of *things* — what Marxists call the 'means of production'. In contrast Nitzan and Bichler focus on the ownership of *institutions*. This is the key to connecting income class to hierarchical power.

My reasoning is best understood by a thought experiment. Imagine you buy all the shares of a corporation. As the sole owner, you now command the corporate hierarchy. In effect, you *purchased* hierarchical power. Once in command of the hierarchy, you have the power to distribute resources within it. You can divide the firm's income stream, keeping some for yourself and giving the rest to your subordinates. Like the feudal king, your income gets its own *class*. As owner, you earn *profit* (capitalist income). Those you command earn *wages* (labor income).

This reasoning leads to a simple model of how capitalist income might relate to hierarchical power. Shown in Figure 4, a single owner commands the hierarchy, and uses his/her authority to divide the firm's income stream. The owner earns capitalist income. Everyone else earns labor income. Expanding

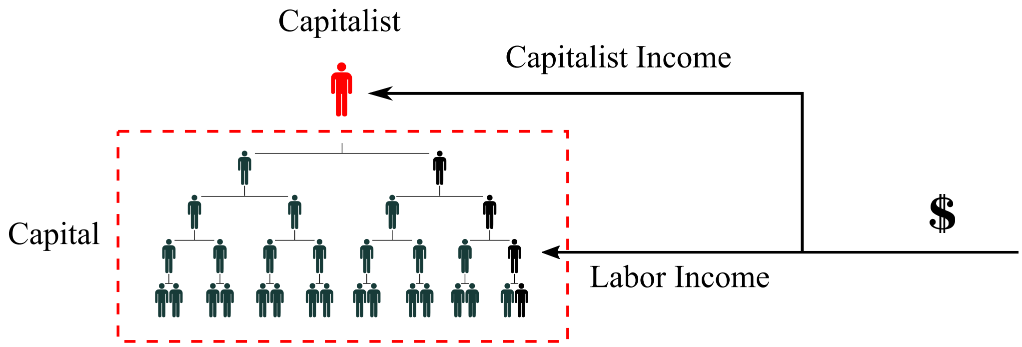

Figure 4: A Sole-Ownership Model of Capitalist Income in a Hierarchy

This figure shows how class-based income might relate to hierarchical rank. We suppose that a capitalist is the sole owner of a firm. This gives the capitalist the legal right to command the firm hierarchy. From this position of power, the capitalist divides the firm income stream and pays himself/herself capitalist income (profit). Everyone else earns labor income. Expanding on Nitzan and Bichler's (2009) concept of 'capital as power', I treat 'capital' as the commodification of the owner's hierarchical power.

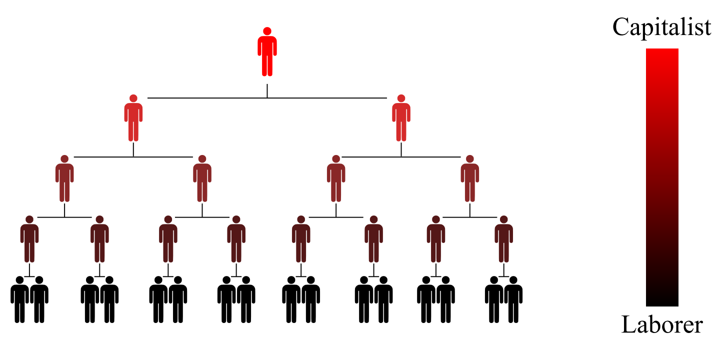

Figure 5: A Gradient Model of Capitalist Income in a Hierarchy

This figure shows a gradient model of class-based income. Ownership is distributed among many individuals but remains connected to hierarchical power. Top-ranked individuals have large ownership shares, while bottom-ranked individuals have small ownership shares. Thus capitalist income fraction increases as a function of hierarchical power. I call this the 'capitalist gradient hypothesis'.

on Nitzan and Bichler's concept of 'capital as power', I represent capital as the commodified ownership of the hierarchy.

A Capitalist Gradient Hypothesis

Our model in Figure 4 is intuitive, but likely too simple. The problem is that there is only a *single* owner. In modern firms, *partial* ownership is the norm. This means ownership is divided among many people.

Does partial ownership mean capitalists no longer control the corporate hierarchy? Berle and Means (1932) thought so. They argued that diffuse ownership caused capitalists to cede control to professional managers. The problem with Berle and Means' 'separation thesis' is that it assumes a *dichotomy* between owners and non-owners. But the truth is that accounting practices have become more complex. Many owners now pay themselves a salary — a non-ownership income. And many employees earn income from stock options — a form of ownership income.

Instead of a dichotomy, what if there is a *gradient* of ownership within firm hierarchies? This would look like Figure 5. Here the firm has many owners. But ownership is still related to hierarchical power. Those at the top have a large ownership stake while those at the bottom have a small one. With this spread of ownership comes a spread of capitalist income. Those at the top still earn mostly capitalist income, and those at the bottom still earn mostly labor income. But in between, the lines are blurred. In short, *class* is (at least in part) a function of hierarchical power. I call this the 'capitalist gradient hypothesis':

Capitalist Gradient Hypothesis: The portion of individuals' income that comes from ownership is proportional to hierarchical power (as measured by Eq. 1).

We can interpret this hypothesis a few different ways. First, we can apply it to a *single* firm. But this is realistic only for firms that are fully employee owned. Such firms do exist, but are not the norm. Second, we could apply the capitalist gradient hypothesis to employee stock ownership plans. These give *partial* ownership to a firm's employees. The problem is that employee ownership makes up about 4% of total US market capitalization.[4] Thus it is not the main source of capitalist income.

I interpret the capitalist gradient hypothesis at the *societal* level. I admit that the ownership structure of any given firm is complex. I also admit that

[4] In 2017, employee ownership plans had total assets of roughly \$1.3 trillion (NCEO, 2017), while total US market capitalization was roughly \$30 trillion, according to the Russel 3000 index.

individuals earn capitalist income from a variety of firms. But at the societal level, I hypothesize that being a capitalist is a function of hierarchical power. That is, those who earn a large portion of their income from ownership still tend to have a high rank in a *single* firm.

To make this thinking concrete, we can look at the Forbes 400 — the 400 wealthiest individuals in the US. True, most of these individuals own stocks in many different companies. But few earned their wealth through diversified investments. Many (if not most) earned their wealth by commanding a *single* large firm (Fix, 2018c). Think Bill Gates and Microsoft, Jeff Bezos and Amazon, Mark Zuckerberg and Facebook. In other words, these individuals wield great hierarchical power within a single firm. The idea is that the capitalist income fraction of these individuals is statistically related to their hierarchical power in a single firm.

2.5 A Unified Theory of Income and Class?

I propose the power-income and capitalist gradient hypotheses as a way to unify the study of income distribution. They tie both income *size* and income *class* to hierarchical power. Note that these hypotheses are framed as *correlations*, not causal relations. I hypothesize that both income size and capitalist income fraction are *proportional* to hierarchical power. While this may be a causal relation, the goal of this paper is to look for *correlation* only. If we find a strong correlation, then future research can unpack the cause(s).

The remainder of the paper tests the power-income and capitalist gradient hypotheses in the United States. Section 3 outlines how I define and measure class. Section 4 tests these hypotheses using a case study of American CEOs. Section 5 tests if CEO trends extend to the general public.

3 Methods: Classifying Income

I discuss here how I classify income to test the capitalist gradient hypothesis. I first outline *ideal* ways of defining income classes (Section 3.1). I then discuss the *actual* measures used in this paper (Section 3.2). To work with the available data, I use and compare several non-ideal measures of capitalist income. As such, empirical results should be considered preliminary.

3.1 What is Capitalist Income?

How we classify income depends on our ideas about property and ownership. I review here two ways of classifying income — one that makes sense from a neoclassical standpoint, and one that makes sense if we treat ownership as a tool for power.

In both approaches, *labor* income is the same. It is income that does not come from ownership. The sticking point is *capitalist* income. Does capitalist income come from *any* form of ownership. Or just *some* forms? The answer depends on our preconceptions about property. If, like neoclassical economists, we think property is a thing that produces value, then all ownership is the same. We should use the class system in Table 1 and treat all property income as capitalist.

The problem with this system is that it mixes two forms of ownership — *scalable* and *non-scalable*. Corporate ownership is *scalable*. Corporations range from tiny shell companies to behemoths like Walmart. They can be any size. But by a quirk of the law, this is not true for proprietor and rental ownership. They are *non-scalable*. By definition, rent can flow only to unincorporated individuals. And proprietors are mostly the self-employed. If either a landlord or a proprietor grows their business, they will incorporate and their income will be reclassified as *profit*. By legal quirk, landlord and proprietor forms of ownership are inherently small scale.

Why does this matter? In neoclassical theory it doesn't. But in my theory it does. I propose that capitalists are not simply those who own *property*. Instead, capitalists are those who own *hierarchy*. This means we want to distinguish between *corporate* and *non-corporate* ownership. Because corporations can be any size, corporate owners can control hierarchies. According to my definition, this means they are *capitalists*. But non-corporate owners are inherently small-scale, so they *cannot* control hierarchies. Thus we give them a separate category of small-scale ownership.

This three-class system is shown in Table 2. It is my preferred class system for testing the capitalist gradient hypothesis. Unfortunately the available empirical data does not (for the most part) fit cleanly into these categories. Thus I am forced to use the non-ideal measures shown in Table 3.

3.2 Empirical Measures of Class-Based Income

I outline here the empirical measures of class income that I use to test the capitalist gradient hypothesis (Table 3). For detailed sources and methods, see the

Table 1: Income Classes if all Owners are Capitalists

Income Type	Symbol	Definition	Composition
Labor Income	L	monetary returns to *non-owners*	wages/salaries + pensions
Capitalist Income	K1	monetary returns to *owners*	distributed corporate profit + interest + rents + proprietor income + capital gains on all property

Table 2: Income Classes if Only Corporate Owners are Capitalists

Income Type	Symbol	Definition	Composition
Labor Income	L	monetary returns to *non-owners*	wages/salaries + pensions
Small-Scale Owner Income		monetary returns to *non-corporate* ownership	rents + proprietor income + capital gains on rental or proprietor property
Capitalist Income	K2	monetary returns to *corporate* ownership	distributed corporate profit + interest+ capital gains on corporate equity and bonds

'Distributed corporate profits' are paid to individuals. This includes dividends from M corporations and profit from S corporations.

Appendix.

Measuring the Capitalist Income of US CEOs

In Section 4, I conduct a case study of American CEOs. I use CEOs because we can estimate their hierarchical power from firm size alone. The difficulty is that data on CEO pay excludes some forms of capitalist income. Firms report only the income that is considered *employee compensation*. This includes CEOs' income from stock options, but excludes income from their personal investments. Still, stock options are a significant source of capitalist income.

Stock options allow an employee to buy corporate stock at a fixed price. The goal is to 'exercise' the option (buy the stock) when the market price is higher than the option price. The difference between the option value and the market

Table 3: Methods for Measuring Class-Based Income in the United States

Measure	Symbol	Source	Composition	Use
Labor Income	L*	World Inequality Database (WID)	wages/salaries + pensions + 'labor' portion of proprietor income	Fig. 11
Capitalist Income of CEOs	K_{CEO}	Execucomp	realized gains from stock options	Fig. 8, basis for hierarchy model
Capitalist Income (All Ownership)	K1*	World Inequality Database (WID)	distributed corporate profit + interest + rents + 'capital' portion of proprietor income + capital gains on all property	Figs. 12 and 13
Capitalist Income (Corporate Ownership Only)	K2	World Inequality Database (WID)	distributed corporate profit + interest + capital gains on corporate equity	Fig. 13

Notes: Pension income includes employee and employer contributions. It excludes asset income from pension investments. 'Distributed corporate profits' are paid to individuals. This includes dividends from M corporations and profit from S corporations. I focus on distributed corporate profits because I am interested in personal income. The other forms of profit (taxed profit and retained earnings) do not flow directly to individuals. For sources and methods, see the Appendix.

value at the exercise time is call the *realized gain*. This gain is taxable income (Hopkins and Lazonick, 2016).[5] Although it is not the complete picture, I use realized gains from stock options to measure the capitalist income of CEOs.

Measuring the Distribution of US Income by Class

To measure the distribution of US income by class (Section 5), I use data from the World Inequality Database (WID). The WID data has unparalleled depth, but comes with some caveats.

[5] I consider realized gains from stock options a form of capitalist income. However, the IRS does not. Instead, the IRS treats stock-option income as employee compensation and taxes it at regular income-tax rates (not capital gain rates). However, this classification has changed repeatedly over the last century. Hopkins and Lazonick (2016) nicely document the battle between business and government over how stock-option income should be classified and taxed.

WID income classes (Table 3) do not align with any of my own (Tables 1 and 2). The quirk is that WID divides proprietor income into *capital* and *labor* components. This means *some* proprietor income is classified as labor income and *some* is classified as capitalist income. The rationale for this division is quintessentially neoclassical. It assumes that part of a proprietor's income comes from their property, and part comes from their labor. This method comes from Piketty et al. (2017b), who are the primary source of the WID data. Piketty et al. assign to proprietors the same capital-labor mix as the corporate sector. I think this method is flawed, but I am stuck with the WID divisions.

The WID labor income series L* (Table 3) is like my definition L (Table 1), but with some proprietor income mixed in. Similarly, the WID capitalist income series K1* is like my definition K1, but with some proprietor income mixed in. When possible, I construct my own measure of capitalist income, K2 (Table 3). But due to WID data constraints, I can do this only for certain types of analysis.

Comparing Different Measures of Capitalist Income

As shown in Table 3, I use and compare different measures of capitalist income throughout the paper. This is by necessity. The available data relating hierarchical power to income and class is scarce. To make progress, I use measures of class-based income that are ideal , and I make comparisons between data that are not perfectly compatible. Given the coarseness of these methods, we should consider the results preliminary. Throughout Sections 4 and 5, I alert the reader to the different measures that are being compared.

4 A Case Study of US CEOs

To study the relation between income, class, and hierarchical power, I conduct a case study of US CEOs. I focus on CEOs because their role as corporate commander allows a shortcut for measuring their hierarchical power (Section 4.1). Instead of needing the whole chain of command, we can estimate CEO hierarchical power from *firm size alone*. Given the paucity of data on firm hierarchy, this is an empirical boon. I use CEO data to test both the power-income hypothesis (Section 4.2) and the capitalist gradient hypothesis (Section 4.3).

4.1 Measuring the Hierarchical Power of CEOs

To measure hierarchical power, we usually need to know a firm's command structure. But this information is proprietary, and economists have spent little time

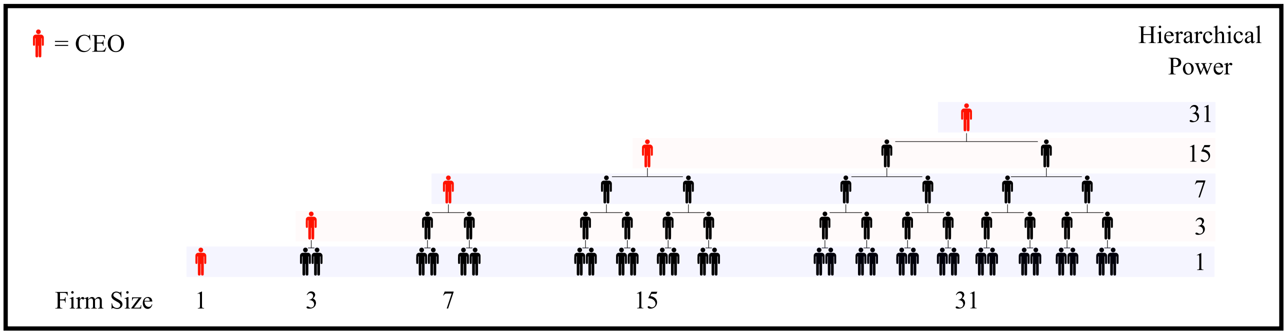

Figure 6: Using Firm Size to Measure the Hierarchical Power of CEOs

This figure illustrates how we can use firm size to measure the hierarchical power of CEOs. Each hierarchy represents a different firm, with the CEO at the top (red). If firm size is x, each CEO has $x-1$ subordinates. Since hierarchical power equals the number of subordinates plus one, CEO hierarchical power is equal to firm size x.

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gathering it. Fortunately, if we focus on the CEOs, there is a shortcut for measuring hierarchical power.

We assume CEOs command the corporate hierarchy. This means every other member is a subordinate. Since hierarchical power is one plus the number of subordinates, the hierarchical power of a CEO equals the *size of the hierarchy* (Fig. 6). Thus we can estimate the hierarchical power of a CEO from firm size alone.

$$\text{CEO Hierarchical power} = \text{firm size} \tag{2}$$

4.2 Hierarchical Power and Pay Among US CEOs

I test here if CEO income is proportional to hierarchical power (the power-income hypothesis, Section 2.3). I look for the same power-income relation that we found in firm case studies (Fig. 3). Quantitatively, this was

$$I_R \propto P^{\beta} \tag{3}$$

where I_R is relative income within the firm, P is hierarchical power, and β is a regression parameter. I call β the 'power-income exponent', since it measures how rapidly income increases with hierarchical power. In firm case studies, I measured I_R as income relative to the bottom hierarchical level. I would do the same for CEOs, but we lack data for the income of bottom-ranked individuals in

CEOs' firms. Instead, I measure CEOs' income relative to the average pay within the firm. I call this the 'CEO pay ratio' for short.

Results are shown in Figure 7. I find that CEO pay strongly correlates with hierarchical power (Fig. 7A). While I would like to claim this is a new discovery, it is not. A half-century ago, David Roberts (1956) found that CEO pay increases with firm size. While Roberts did not frame this relation in terms of hierarchical power, his work prompted Herbert Simon (1957) to create the first model of how hierarchy affects pay. In hindsight, Roberts' discovery was some of the first evidence that hierarchy shapes income. His finding holds true to this day.

On that note, Figure 7B shows that from 2006–2016, the CEO power-income relation was relatively constant. The method works as follows. In each year, I regress the power-income exponent β on the CEO data. The histogram then shows the distribution of β. From 2006–2016, the relation between CEO pay and hierarchical power was within the range $0.4 < \beta < 0.5$.

Table 4: Power-Income Exponent Among US CEOs and Within Case-Study Firms

	Power-Income Exponent (β)
US CEOs	0.45
Case-Study Firms (Fig. 3)	0.34

Notes: β is defined by Eq. 4. It measures how rapidly income increases with hierarchical power.

With new data, we naturally want to see how it compares with existing evidence. To that end, Table 4 compares the power-income relation among CEOs to one found in the case-study firms (3). Although β estimates differ between the two samples, this may not be statistically significant. The case-study sample is small (6 firms), which creates sampling uncertainty. To judge the data fairly, we need to account for this uncertainty.

To do this, we analyze the data at the *firm level*. For each case-study firm, we regress the power-income exponent β. For each CEO observation, we estimate β *within* the CEO's firm using:

$$\beta = \frac{\log(I_R)}{\log(P)} \tag{4}$$

This is Eq. 3 rearranged for β. Rather than a regression on many data points, Eq. 4 uses a *single* data point to estimate the power-income relation within each

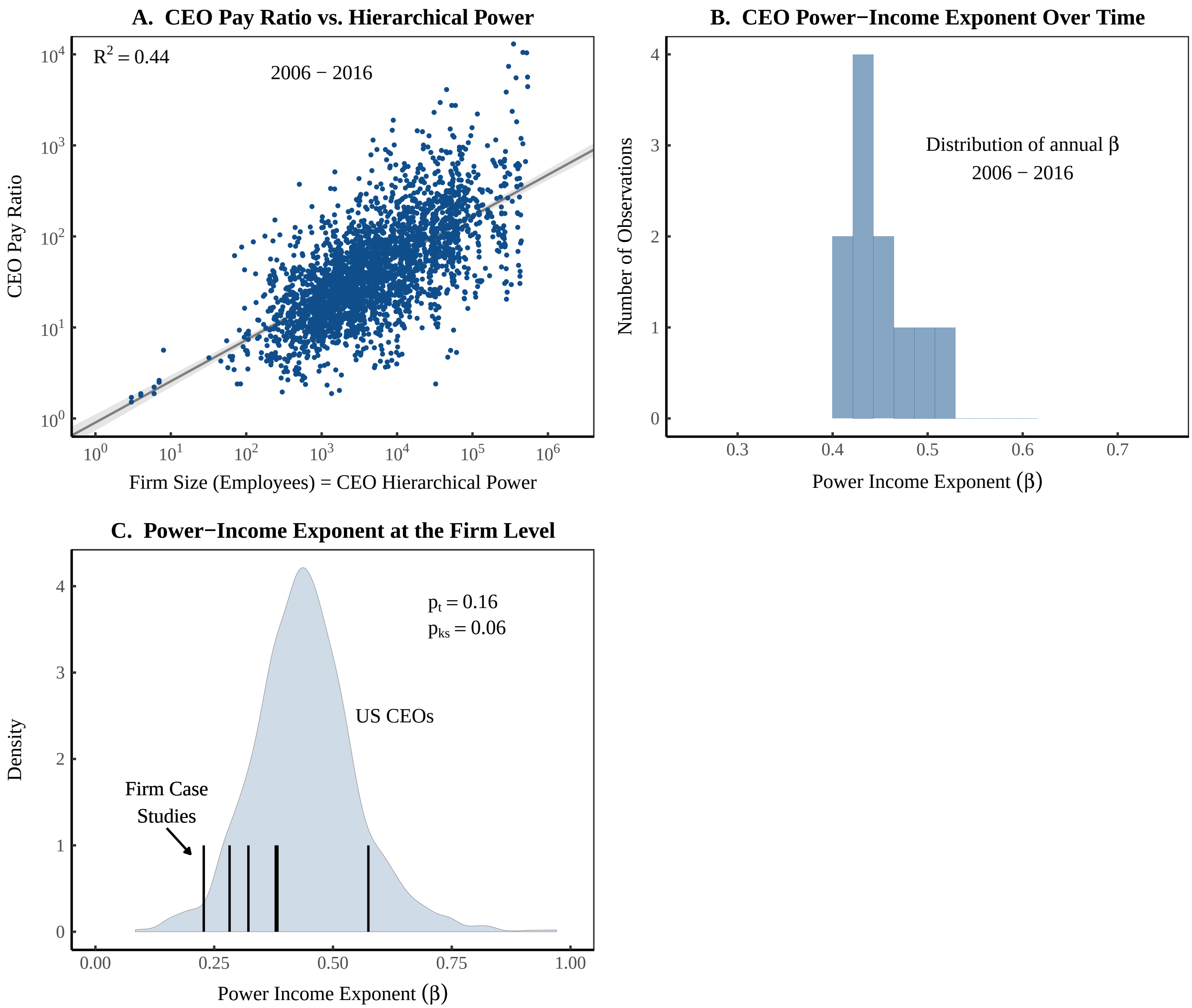

Figure 7: Income vs. Hierarchical Power Among US CEOs

This figure shows the relation between income and hierarchical power among US CEOs. The hierarchical power of CEOs is measured by firm size. Panel A plots the CEO pay ratio (CEO pay relative to the firm mean) against CEO hierarchical power. Each data point represents a CEO. Observations cover the years 2006–2016. The line indicates a log-log regression. Panel B shows the distribution of annual power-income exponents over this period (Eq. 3). Panel C shows firm-level power-income exponents. Vertical lines indicate β for each case-study firm (Fig. 3). I compare this to the distribution of firm-level β estimates for US CEOs (Eq. 4). p_t indicates the p-value of a t-test on these two samples. p_{ks} indicates the p-value of a Kolmogorov–Smirnov (KS) test. Both suggest no significant difference (at the 5% level) between CEO and case-study samples. For data sources and methods, see the Appendix.

CEO's firm. We use the CEO pay ratio as our measure of relative income (I_R), and firm size as our measure of CEO hierarchical power (P).

Figure 7C shows the results of this analysis. The density curve shows the distribution of β within CEOs' firms. Vertical lines indicate β for *each* case-study firm. To compare the two samples, I use a t-test and KS-test. These tests assume both samples come from the same population (the *null hypothesis*), and return the probability of drawing two samples as *different* as our given samples. According to the t-test, this probability is 16%. According the KS-test, it is 6%. These probabilities are too large to reject the null-hypothesis. In other words, the power-income relation among CEOs is statistically consistent with the relation in firm case studies.

To summarize, I find strong evidence that CEO income increases with hierarchical power. This relation is roughly constant over a ten-year period, and is statistically consistent with the relation found in case-study firms.

4.3 Hierarchical Power and Capitalist Income Among US CEOs

I now move from looking at CEOs' income *size* to looking at the capitalist *portion* of their income. I test if this portion increases with hierarchical power (the capitalist gradient hypothesis, Section 2.4).

I define the capitalist income of CEOs as the realized gains from stock options. I then measure the capitalist *fraction* of total income:

$$\text{Capitalist Fraction of CEO Income} = \frac{\text{Realized Gains from Stock Options}}{\text{Total Compensation}} \tag{5}$$

As shown in Figure 8, I find that the capitalist fraction of CEO income increases with hierarchical power. Quantitatively, the relation can be modeled as

$$K_{\text{frac}} = \kappa \log(P) \tag{6}$$

where K_{frac} is the capitalist fraction of income, P is hierarchical power, and κ is a free parameter. I call κ the 'capitalist gradient slope', as it measures how rapidly the capitalist fraction of income increases with hierarchical power.

The raw correlation between the capitalist fraction of income and hierarchical power is not large ($R^2 = 0.03$), but is highly statistically significant ($p < 10^{-10}$). In other words, the trend is important but noisy. The relation is also consistent over time. Figure 8B regresses κ onto annual CEO data and plots the

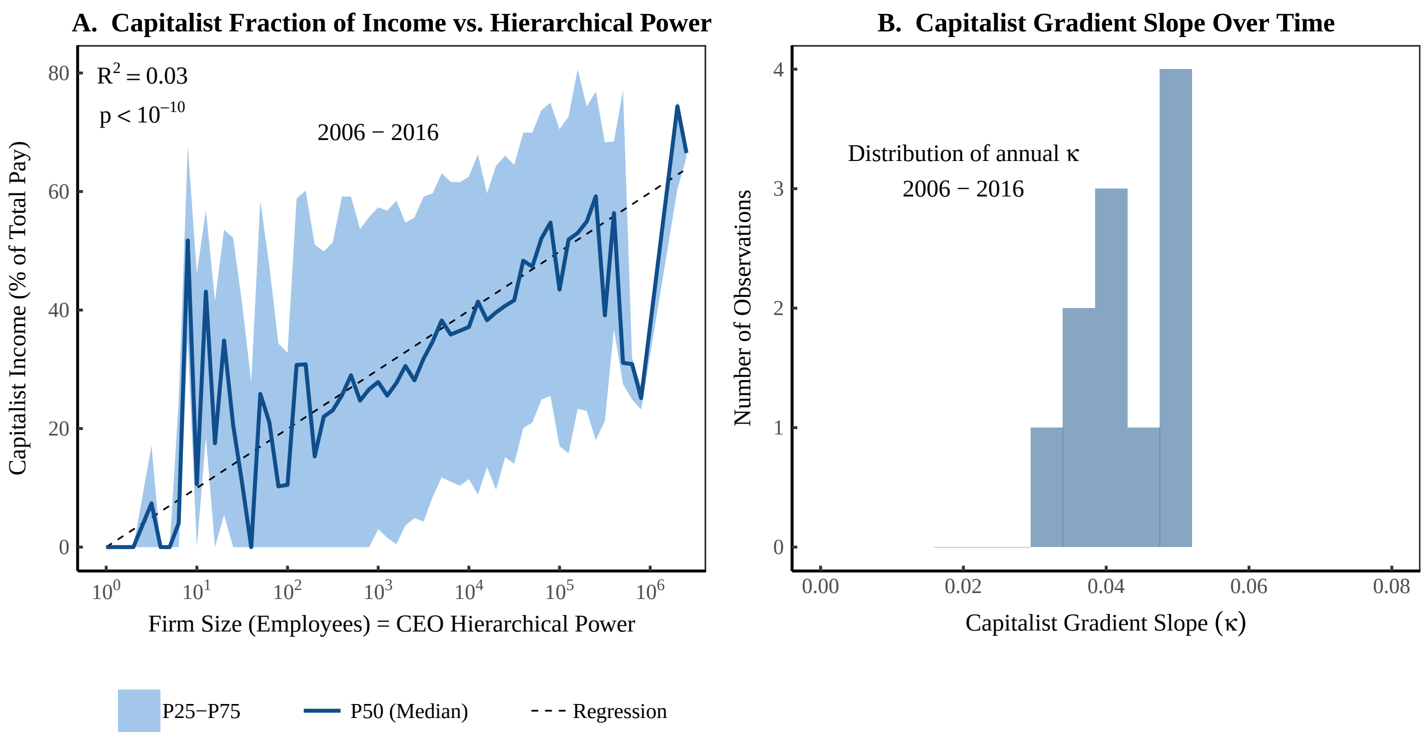

Figure 8: Capitalist Fraction of Income vs. Hierarchical Power Among US CEOs

This Figure shows how the capitalist fraction of CEOs' total income relates to their hierarchical power (measured using firm size). Panel A shows average trends between 2006–2016. Results are for roughly 20,000 US CEOs. The line and shaded region indicate the median and interquartile range (respectively). The dotted line is the regressed trend (a log-linear regression with y-intercept fixed to 0). Panel B shows the distribution of the capitalist-gradient slope κ (Eq. 6) over the years 2006–2016. For data sources and methods, see the Appendix.

distribution over 2006–2016. As with the power-income relation (Fig. 7C), the capitalist gradient relation is consistent over time.

To summarize, I find that CEOs with more hierarchical power tend to earn a larger portion of their income from ownership. This suggests that, at least among CEOs, being a capitalist is a function of hierarchical power.

4.4 CEO Case-study Summary

The evidence indicates that both the income *size* and income *class* of CEOs are functions of hierarchical power. This supports the power-income and capitalist gradient hypotheses. But since CEOs are a unique group, we must be careful about inferring too much from them alone. To draw more general conclusions, we need to test if CEO trends extend to the general public. Because we lack data on hierarchical power for the general public, I use a model to conduct this test (Section 5).

5 From CEOs to the General Public: Extending the Evidence

To test if there is a general relation between income, class, and hierarchical power in the United States, I propose the following method:

Assume CEO trends extend to the general public → Model the implied distribution of income → Compare to the actual distribution of income

This test works by inference. We use a model (Section 5.1) to predict the distribution of US income that should occur if CEO trends are *also* found among the general public. If the model's predictions are correct, we can *infer* the general relation between income, class, and hierarchical power (without actually observing it). I find that the model's predictions are generally consistent with US data. Within its range of uncertainty, the model correctly predicts the size distribution of US income by class (Section 5.2). It also predicts, with reasonable accuracy, the relation between income *size* and income *class* (Section 5.3). This suggests that CEO trends extend to the general public. I discuss the consequences in Section 6.

5.1 Using A Model to Extrapolate CEO Trends

To extrapolate the CEO evidence to the general public, I use a numerical model developed in Fix (2018c). This 'hierarchy model' takes the CEO data as an input (as well as the firm case-study data) and then predicts the distribution of income that should occur if CEO trends extend to the US public. I use the word 'predict' here sincerely. The model's parameters are determined entirely by *firm-level* data. The model then predicts the *macro* distribution of income — an independent phenomenon. I do not 'tune' the model's parameters to produce desired results.

I discuss here the hierarchy model's *conceptual* structure. See the Appendix and Fix (2018c) for technical details.

The Conceptual Structure of the Hierarchy Model

The hierarchy model takes what we know from CEOs, and fills in the gaps for the rest of the population. The model has three conceptual steps, shown in Table 5. In Step 1, the model simulates the hierarchical structure of CEOs' firms. In Step 2, the model generalizes this simulation to a more representative size distribution of firms. In Step 3, the model simulates class-based income. The result is a predicted distribution of US income by class.

Step 1: Simulate the hierarchical structure of CEOs' firms. The model begins with what we know. We have data on firm size and CEO pay for a sample of US firms. We also know the hierarchical structure of six case-study firms (Fig. 3). The model combines these two datasets to simulate the hierarchical structure of CEOs' firms. To simulate the *employment* hierarchy, the model assumes that CEOs' firms have the same hierarchical 'shape' as case-study firms. The model then uses the CEO pay ratio to infer the pay hierarchy within each CEO's firm. The result is a simulation of the hierarchical structure of CEOs' firms.

Step 2: Generalize the simulation to the US firm population. We cannot end with Step 1 because the sample of CEOs' firms is *size-biased* — it contains more large firms than the US firm population. To draw accurate conclusions, we must extend our simulation to the correct size distribution of firms. We do this by analyzing our simulation of CEO firms. We observe how firm mean pay is distributed, and how hierarchical pay changes with firm size. We fit functions to these trends. Armed with these functions, we create a *new* simulation that replicates the US firm size distribution. We then use this general model to predict the distribution of US *personal* income.

Step 3. Simulate class-based income. Having simulated personal income, the last step is to divide individuals' income into class components. We assume that the relation between the capitalist fraction of CEOs' income and their hierarchical power (Fig. 8A) extends to the general public. We model the capitalist fraction of individuals' income using Eq. 6, setting the capitalist gradient slope (κ) to the one found in CEOs. We then define the labor fraction of individuals' income (L_{frac}) as the *complement* of the capitalist fraction:

$$L_{\text{frac}} = 1 - K_{\text{frac}} \tag{7}$$

The goal here is to simulate a two-class system of income, as in Table 1.

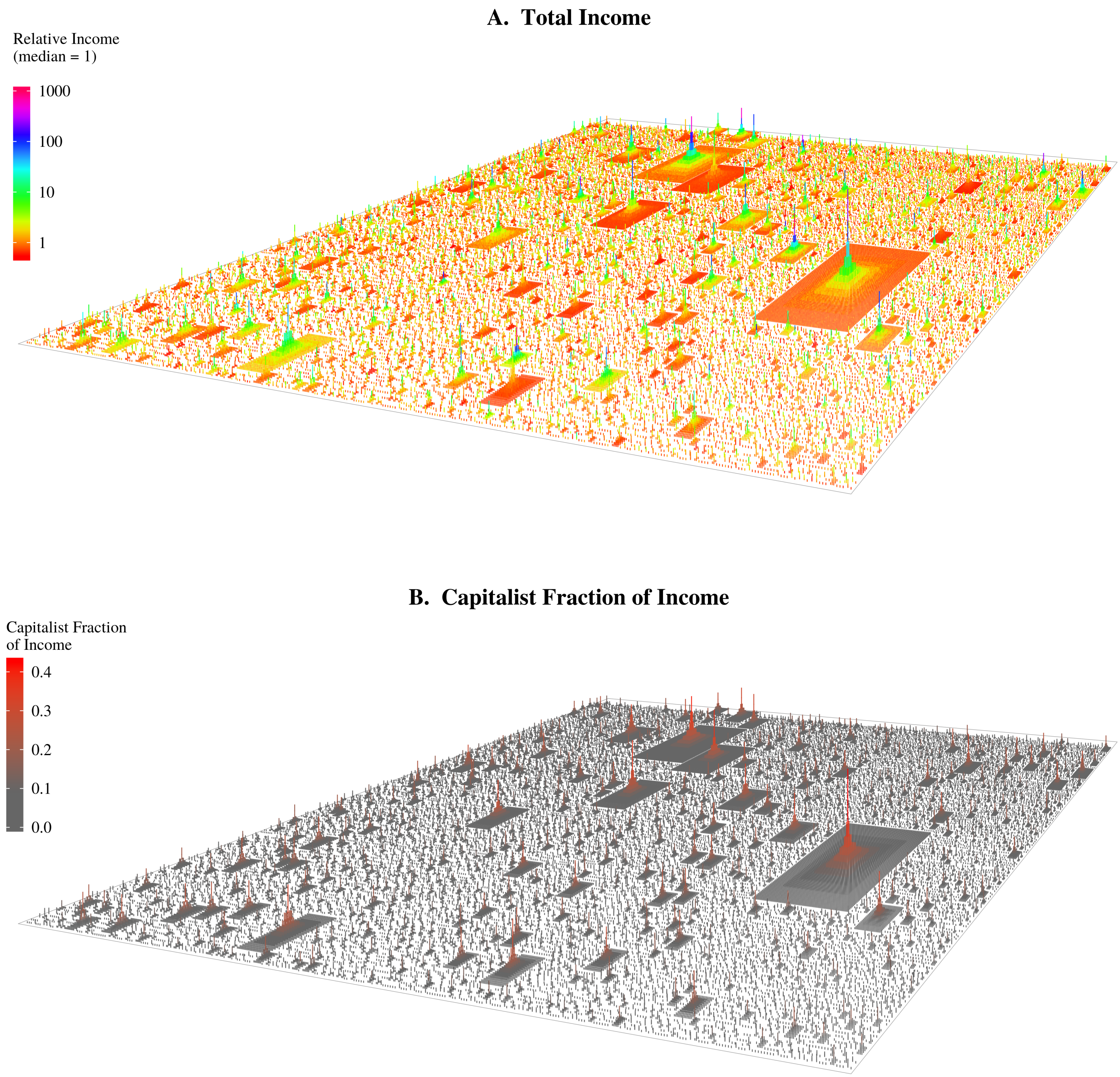

Figure 9: A Landscape View of the US Hierarchy Model

This figure visualizes the US hierarchy model as a landscape of three-dimensional firms. The model extrapolates CEO data to create a simulation of how income and class relate to hierarchical power in the United States. Each pyramid represents a single firm, with size indicating the number of employees and height corresponding to the number of hierarchical levels. Panel A uses color to indicate income relative to the median. Panel B uses shades of red to indicate the capitalist fraction of individuals' income. This visual shows 20,000 firms. The actual model uses 1 million firms to simulate the US firm population.

Table 5: The Hierarchy Model's Conceptual Structure

	What We Know	What We Do Not Know	Simulation
Step 1	• CEO pay-ratio and firm size for a large sample of US firms • Hierarchical structure of 6 case-study firms	The hierarchical employment and pay structure *within* CEO firms	Simulate the hierarchical structure of 'CEO firms'*
Step 2	Simulated hierarchical structure of CEO firms	How this size-biased simulation of CEO firms generalizes to the US firm population	• Simulate the hierarchical structure of the *whole* US firm population • Predict the distribution of personal income
Step 3	Simulated personal income US population	Class-based income of individuals	Simulate class-based income using the CEO capitalist gradient
Model Output	• Simulation of US personal and class-based income distribution • Simulated relation between income size, income type, and hierarchical power		

* 'CEO firms' refers to the sample of US firms for which we know the CEO pay ratio and firm size (Fig. 7)

Model Output. Figure 9 visualizes the model's output as a landscape, with firms displayed as pyramids. We can see the main features of the model here. Income increases rapidly with hierarchical rank (Fig. 9A), as does the capitalist fraction of income (Fig. 9B). We can also see the size distribution of firms. Most firms are small, but there are a few behemoths. It's at the pinnacle of these large firms that we find most top earners and capitalists.

5.2 Model Predictions: The Size Distribution of US Income By Class

I test here the model's predictions for the size distribution of income by class. Figures 10–12 show the results. Each figure compares model predictions to US data over the years 2006–2014. Income distribution metrics are defined in Table 6. Figure 10 shows the distribution of total *personal* income, Figure 11 shows *labor* income, and Figure 12 shows *capitalist* income. A reminder to the reader that the model and the US data use incommensurate definitions of class (see Table 3). The model is based on CEO data, which defines capitalist income in terms of the realized gains on stock options. US capitalist income includes all forms of property income (except a portion of proprietor income).

I first test the model's predictions using three summary statistics of inequality (Figs. 10–12, Panels A–C). I use the Gini index, the top 1% share, and the power-law exponent of the top 1%. Each metric is sensitive to a different aspect of the distribution. The Gini index is sensitive to the *body* of the distribution. The top 1% share is more sensitive to the distribution *tail*. The power-law exponent is sensitive *only* to the distribution tail. When measured with these statistics, the model is quite accurate. The average error is less than 11% (Table 7).

I also test the model by visualizing the income distribution (Figs. 10–12, Panels D–G). The probability density (Panel D) visualizes the *body* of the distribution. For all three classes of income, the model reproduces the essential form of the distribution body. I use the complementary cumulative distribution (Panel G) to visualize the behavior of *top* incomes. Again, the model's predictions are roughly consistent with US data. By this, I mean that the US data lies within the model's range of uncertainty.

While the model's predictions are generally consistent with US data, there is one area of failure. The model predicts too few people with small incomes. This is visible in the probability density function (Panel D) as a divergence between the model and the US data as income approaches zero. This divergence is visible in the cumulative distribution (Panel F) as differing y-intercepts between the model and US data.

The prevalence of small *labor* and *personal* income in the US is likely caused by *unemployment* and *non-employment*. Many US individuals do little paid work, either by choice or by circumstance. This non-employment is not part of the hierarchy model. Why? The model extrapolates *corporate* data, which is biased towards the full-time employed. Thus the model's under-prediction of small incomes is likely due to factors it excludes.

For *capitalist* income, the problem is slightly different. The model assumes that the capitalist fraction of individual income is an *exact* function of hierarchical power. This means that individuals with the same number of subordinates have *exactly* the same capitalist income fraction. This leaves no room for small variations in rates of return or the size of investment. For small capitalist incomes, this variation likely dominates income dispersion and cannot be neglected.

To summarize, the model's predictions for the US distribution of income by class are generally consistent with the US data. The only consistent failure is

Table 6: Definitions of Income Distribution Metrics

Metric	**Definition and Sensitivity**
Gini index	A measure of inequality ranging from 0 (perfect equality) to 1 (perfect inequality). The Gini index is sensitive to the *body* of the distribution.
top 1% share	A measure of inequality — the income fraction held by the top 1% of earners. The top 1% share is more sensitive to the distribution *tail* than the Gini index.
power-law exponent (α)	The fitted power-law exponent for the top 1% of incomes. The exponent α indicates that the probability of finding someone with income x is proportional to $1/x^{\alpha}$. The power-law exponent is sensitive only to the *tail* of the distribution. A smaller exponent indicates a 'fatter' tail.
income probability density	Indicates the probability of finding an individual with the given income. Visualizes the *body* of the distribution.
Lorenz curve	Shows the cumulative fraction of income held by individuals with income below the given percentile. A straight line indicates perfect inequality.
cumulative distribution	The curve indicates the fraction of individuals with income *less* than the given x value. Visualizes the *body* of the distribution.
complementary cumulative distribution	The fraction of individuals with income *greater* than the given x value. Visualizes the *tail* of the distribution. When plotted on log-log scales, the slope of the tail relates to the power-law exponent (slope $= \alpha - 1$).

an under-prediction of small incomes. We can plausibly attribute this error to factors that the model excludes.

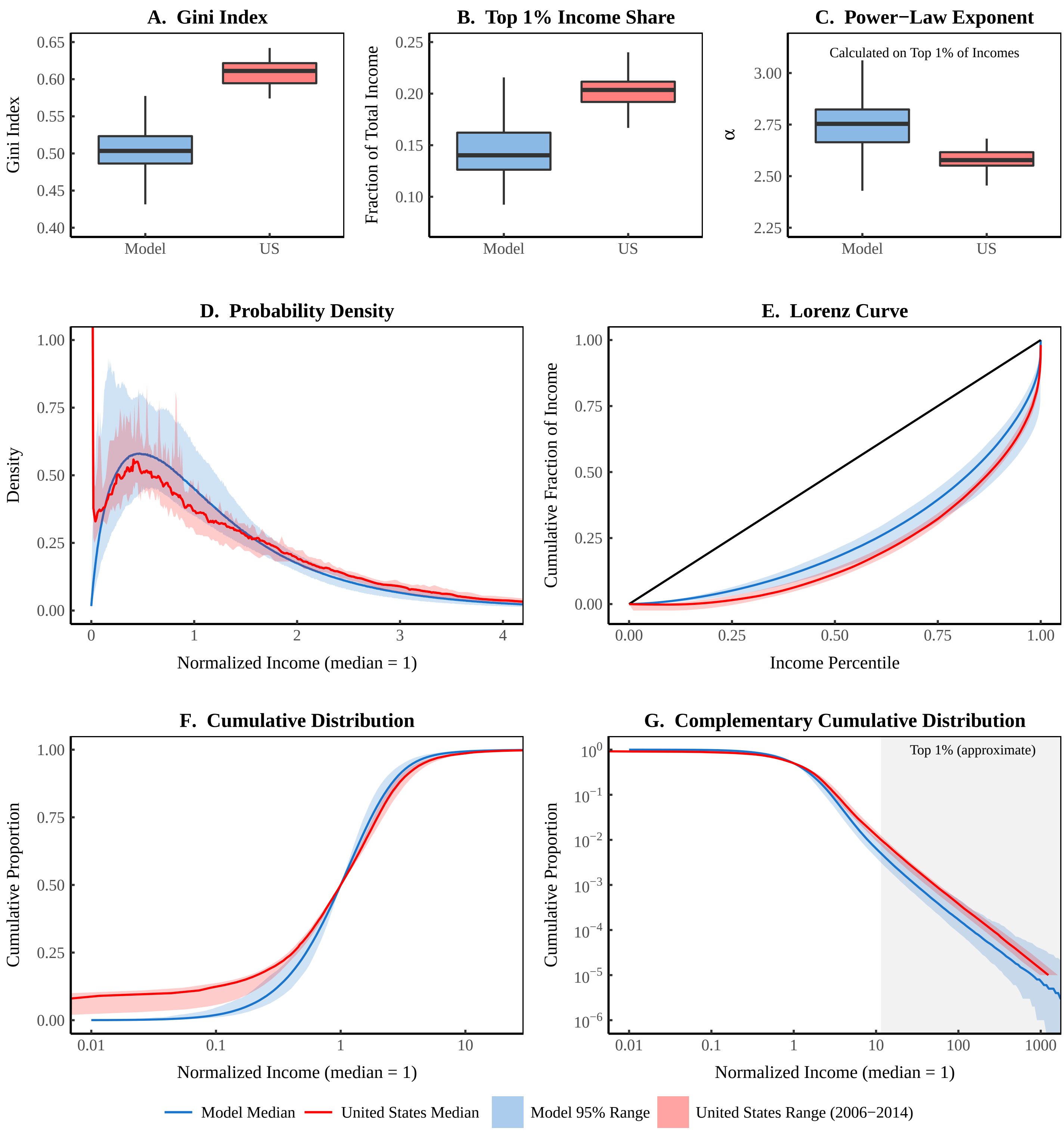

Figure 10: Personal Income Distribution —Model Predictions vs. US Data

This figure shows the distribution of *total personal income*. Each panel compares the hierarchy model's prediction to US data. Metrics are defined in Table 6. Income in Panels D, F and G is normalized so the median is 1. In Panel G, the shaded region shows the approximate threshold for the top 1% of incomes. US data comes from the World Inequality Database. The hierarchy model is stochastic and varies between iterations. I show the model's 95% range. For sources and methods, see the Appendix.

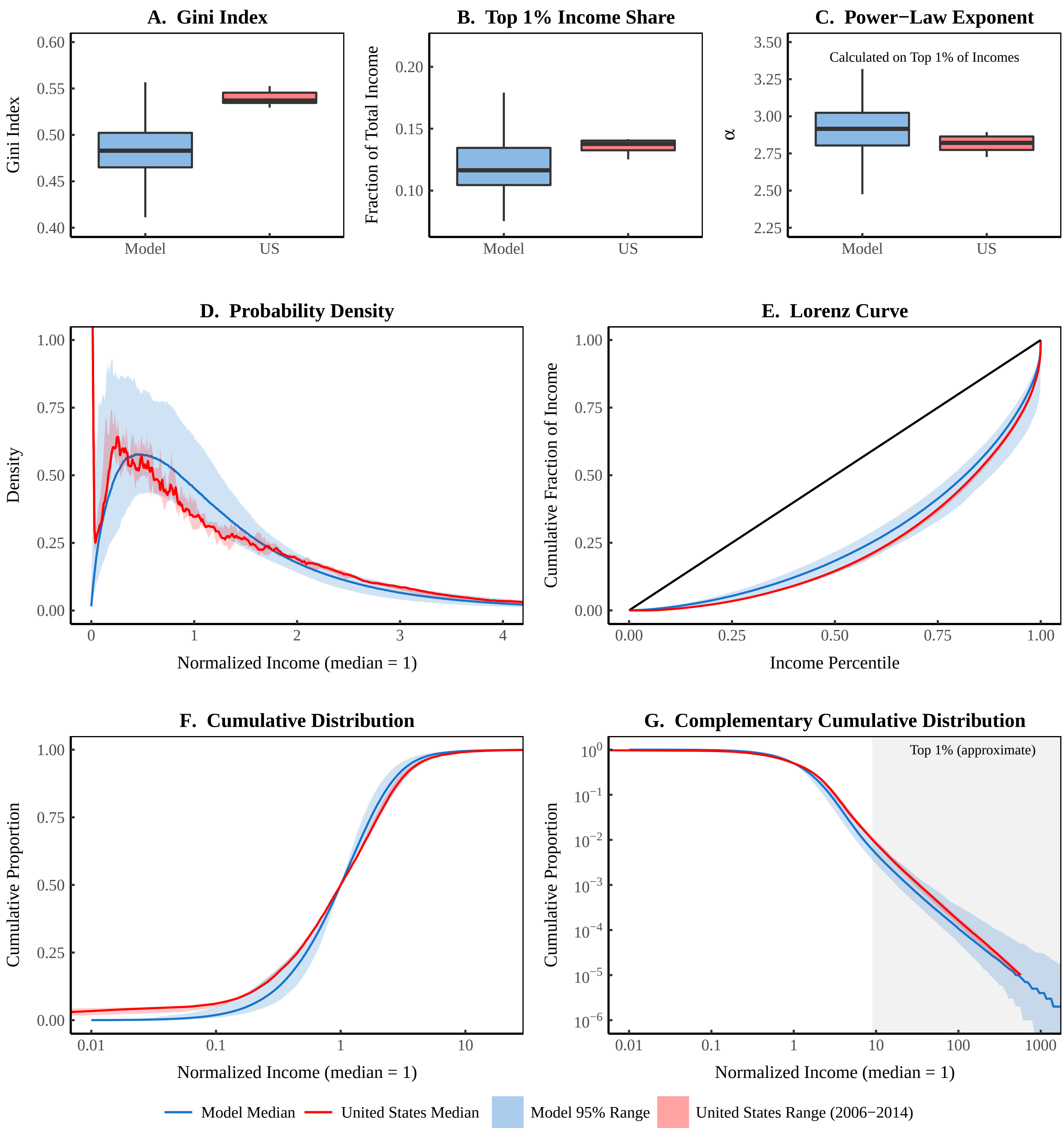

Figure 11: Labor Income Distribution — Model Predictions vs. US Data

This figure shows the distribution of *labor income*. Each panel compares the hierarchy model's prediction to US data. Metrics are defined in Table 6. Income in Panels D, F and G is normalized so the median is 1. In Panel G, the shaded region shows the approximate threshold for the top 1% of incomes. US data comes from the World Inequality Database, using class definitions L1* (Table 3). The hierarchy model is stochastic and varies between iterations. I show the model's 95% range. For sources and methods, see the Appendix.

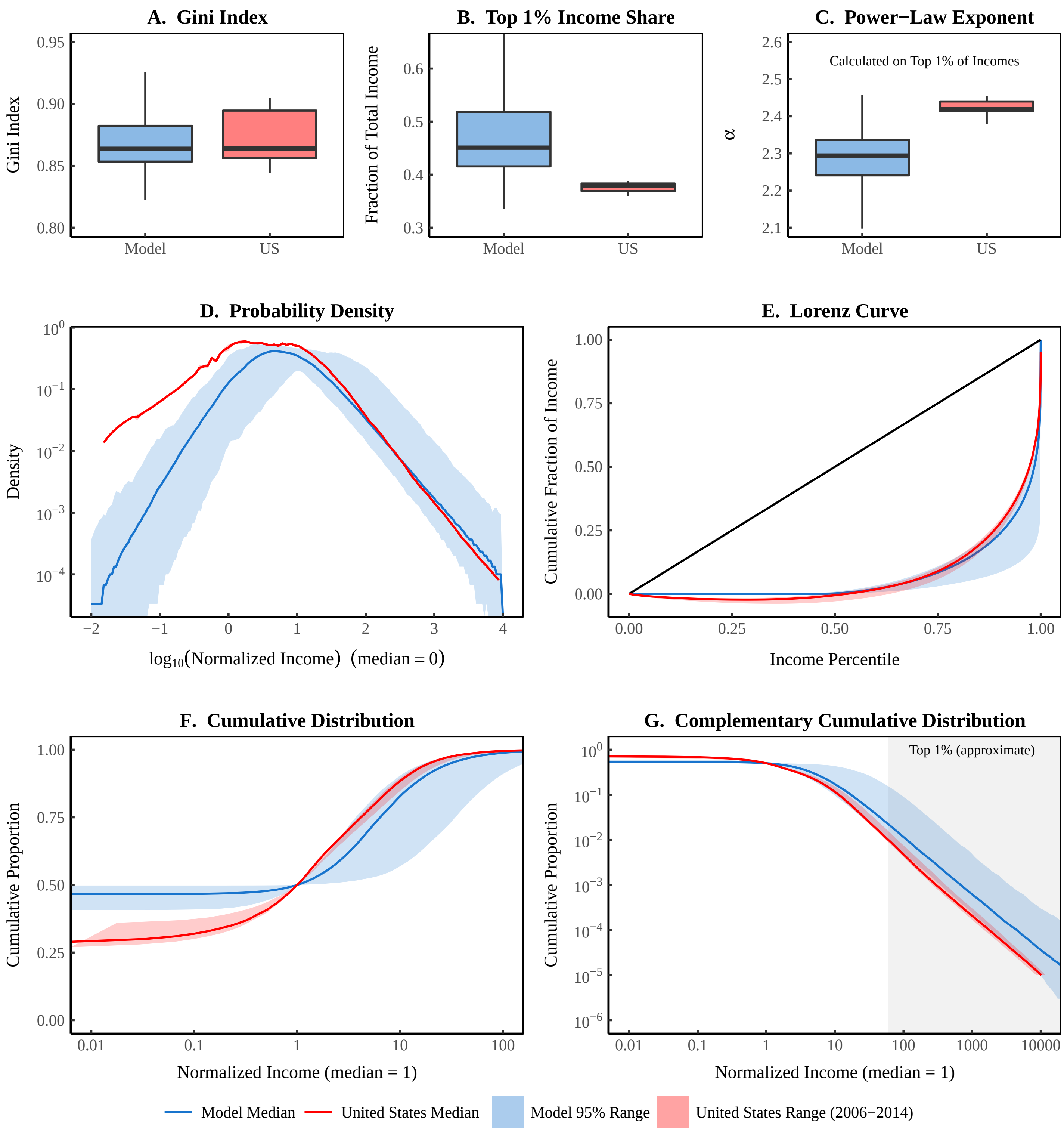

Figure 12: Capitalist Income Distribution — Model Predictions vs. US Data

This figure shows the distribution of *capitalist income*. Each panel compares the hierarchy model's prediction to US data. Metrics are defined in Table 6. Income in Panels D, F and G is normalized so the median is 1. In Panel G, the shaded region shows the approximate threshold for the top 1% of incomes. US data comes from the World Inequality Database, using class definitions K1* (Table 3). The hierarchy model is stochastic and varies between iterations. I show the model's 95% range. For sources and methods, see the Appendix.

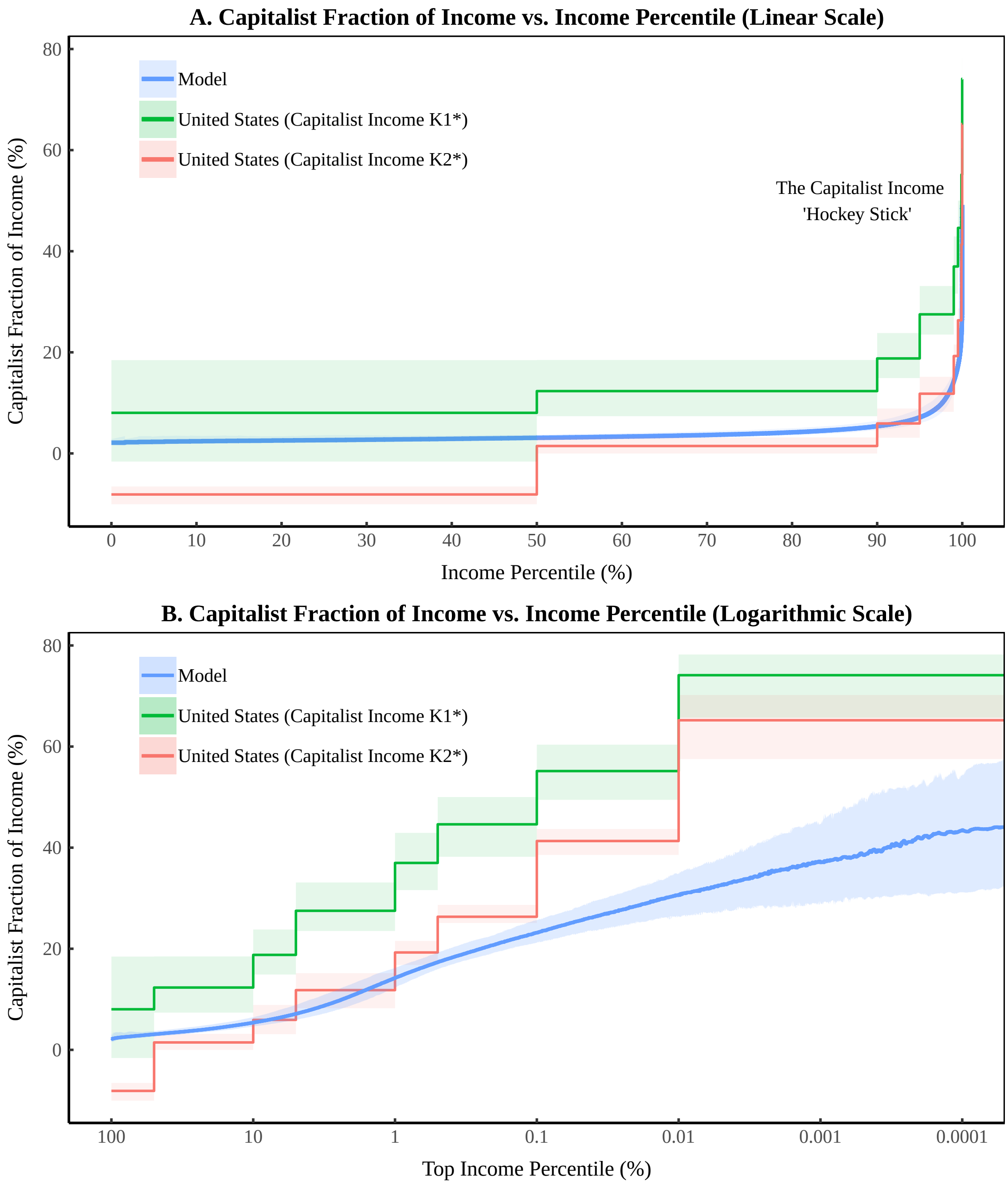

Figure 13: The Capitalist Income Hockey Stick — Model Predictions vs. US Data

This figure shows the capitalist income 'hockey stick' — the relation between the capitalist fraction of income and income percentile. I use two different measures of US capitalist income — K1* and K2* (see Table 3). Panel A uses a linear scale on the horizontal axis. Panel B shows the same data with a reverse logarithmic scale on the horizontal axis. Shaded regions indicate the 95% range of the data. Lines indicate the median. US data covers the years 2006–2014. For sources and methods, see the Appendix.

Table 7: Summary Statistics of Inequality — Model Predictions vs. US Data

Metric	Source	Labor Income	Personal Income	Capitalist Income
Gini Index (mean)	US	0.54	0.61	0.87
	Model	0.49	0.51	0.87
Top 1% Share (mean)	US	0.14	0.20	0.38
	Model	0.13	0.16	0.48
Power Law Exponent (mean)	US	2.82	2.57	2.42
	Model	2.90	2.73	2.28
average model error =10.9%				

This table compares model predictions to US data using the average of three different summary statistics of inequality. For the US, this is the mean between 2006–2014. For the model, it is the mean over many iterations. 'Average model error' indicates the mean percentage error of the model for all the statistics shown in this table.

5.3 Model Predictions: The Capitalist Income 'Hockey Stick'

Next I test the model's predicted relation between income *size* and income *class*. Figure 13 shows the results, plotting the capitalist fraction of income against income percentile. I call this the capitalist income 'hockey stick' because of its similarity to the famous hockey-stick graph showing exploding temperatures in the 20th century (Mann et al., 1999). Here it is capitalist income that explodes among top earners.

In Figure 13, I compare the model to two measures of US capitalist income. I continue using capitalist income K1* (as in Figures 10–12), which includes income from all forms of property. But I also calculate capitalist income K2*, which includes only income from interest and corporate equity (see Table 3).

A reminder to the reader that the model's definition of class income are technically not compatible with the US data. The model is based on CEO data, which measures capitalist income from the realized gains on stock options (Table 3).

Overall, the model's predictions fair well. The model correctly predicts the explosion of capitalist income among top earners. Viewed on a linear scale (Fig. 13A), the model's prediction is very close to US capitalist income K2*. There is a discrepancy among the bottom 50% where the WID data allows negative incomes (from debt). Since the model does not allow negative incomes, it cannot reproduce this behavior. When we view the trends on a logarithmic scale (Fig.

13B), the model still fairs well. But this time we see a discrepancy beginning in the top 0.1%. The model predicts too little capitalist income for these top earners. While not perfect, the model's predictions are consistent with the general form of US data.

Given the model's success, an element of the empirical data becomes more significant. The trend in the capitalist fraction of income is driven by *interest* and *corporate equity* (shown in K2*). We know this because including rent and proprietor income (in K1*) does not significantly change the trend. Why is this important? Because I argued that only *corporate* income should relate to hierarchy (Section 3). Thus, if a trend in capitalist income is due to hierarchy, it should be most visible in corporate data. Now we find that the trend in corporate income is consistent with the trend we expect if hierarchical power relates to income and class among the general public.

5.4 Extending Evidence – A Summary

To summarize, I have used a model to test if the CEO relation between hierarchical power, income, and class extends to the general public. The test works by inference. The model predicts the distribution of income that *should* occur if CEO trends generalize to the US public. We then test this prediction by comparing it to US data. The results are promising. The model's predictions are generally consistent with the US data. This suggests that among the US public, income and class are a function of hierarchical power.

6 Inferences: How the Rich Are Different

We are now in a position to investigate our primary question: *how are the rich different*? Mainstream economists think the rich are more productive. Instead, I have proposed that the rich are different because of their *hierarchical power*. The rich command hierarchies. The poor do not.

We can now use our model to infer if this is true in the United States. To reiterate, the model predicts how income and class should relate to hierarchical power if CEO trends extend to the US public. Having established that this extension is plausible (Section 5), we can now go one step further. We use our model to make the first estimate of how income and class relate to hierarchical power in the United States. The results are shown in Figure 14.

Our inference indicates a startling division in the US population. Or perhaps *chasm* is a better word. The model suggests that the vast majority of Americans

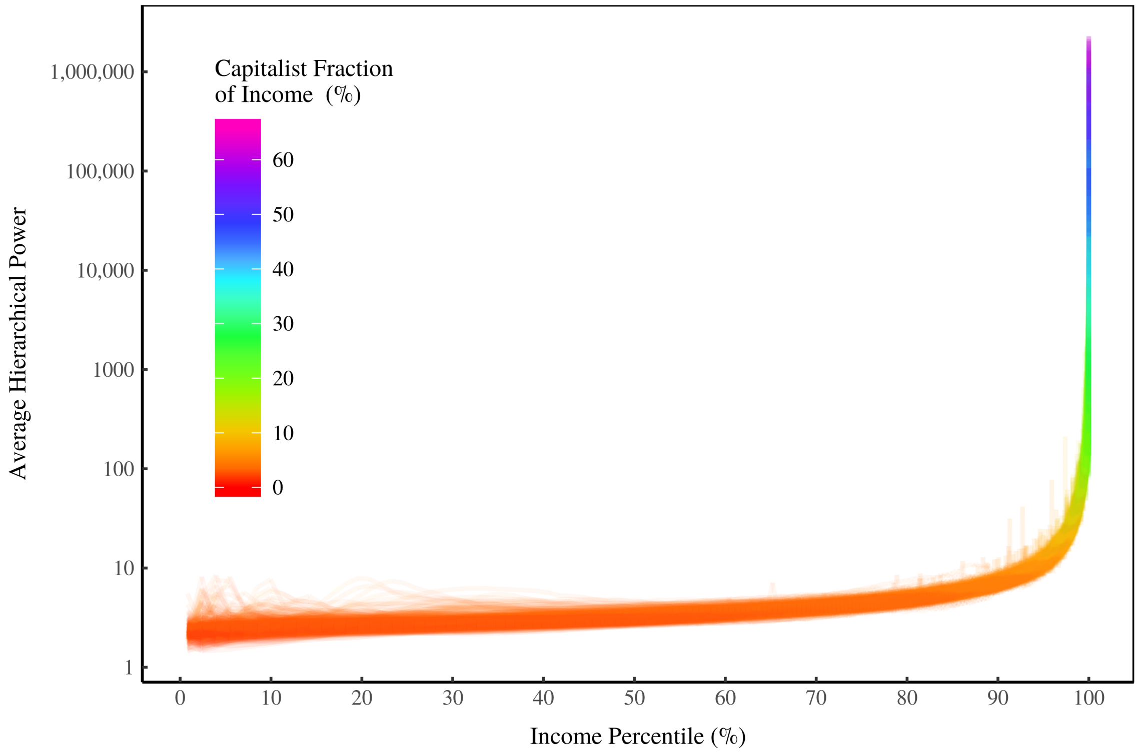

Figure 14: Hierarchical Power — How the Rich Are Different

This figure shows the inferred three-way relation between income size, income class, and hierarchical power among the US public. Results are from the hierarchy model, which extrapolates CEO trends to the general public (Section 5). The plot shows the average hierarchical power of individuals, grouped by income percentile. The average capitalist fraction of income is indicated by color. Lines indicate different model iterations. Note the log scale on the y-axis. The maximum hierarchical power in the model corresponds to a single individual commanding a firm of roughly 2 million employees — the current size of Walmart.

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have virtually no hierarchical power. Yet among the top 1% there is an explosion of hierarchical power — a veritable wall of power. This reinforces Tim Di Muzio's (2015) distinction between '1% and the rest of us'. There seems to be a chasm of hierarchical power separating the two groups. Note that in Figure 14, hierarchical power is plotted on a logarithmic scale. This actually compresses the magnitude of the explosion. According to the model, top earners can have *hundreds of thousands* of times more hierarchical power than the poor.

And it is not just income, but also *class* that changes with hierarchical power. The capitalist fraction of income increases with hierarchical power. In other words, the rich have more hierarchical power, and they are more likely to be capitalists. In short, our inference suggests that in the United States, hierarchical power potentially unifies the study of income and class.

6.1 Open Questions

The inferred relation between income, class and hierarchical power raises many questions for the future. First, we want to know if our inference is *correct*. Does it represent the *actual* relation between hierarchical power, income and class among the US public? Answering this question requires better data. It requires that more researchers be interested in studying the hierarchical structure of firms.

Second, there is the matter of *causation*. The goal of this paper has been to test for a *correlation* between hierarchical power and income size and income class. I have left the study of causation for future research. Unlike neoclassical theory, a power theory of income acknowledges that income is a *social* phenomenon. This means causation is complex. Many different factors are likely at work.

In despotic hierarchies (such as slave plantations), the chain of command may operate through brute force. But in less despotic hierarchies, *ideology* is likely more important. I have argued that the ideology legitimizes both the authority and income of superiors, and gives this income its own class. Understanding the ideologies of power is no small task. Generations of political economists have studied it, with no consensus in sight (Berle and Means, 1932; Brown, 1988; Commons, 1924; Dugger, 1989; Galbraith, 1985; Lenski, 1966; Mills, 1956; Munkirs, 1985; Nitzan and Bichler, 2009; Peach, 1987; Sidanius and Pratto, 2001; Tool and Samuels, 1989; Tool, 2017; Veblen, 1904, 1923; Weber, 1978; Wright, 1979). I leave the difficult task of untangling causation for future research.

Second, there is the question of the 'boundaries' of hierarchy. I have focused here on hierarchy *within* firms. But hierarchies can also extend *between* firms — what Bichler and Nitzan (2017) call "meso hierarchies". This involves the use of partial ownership to wield control over firms. Interesting work on corporate ownership has revealed that investment firms wield a surprising amount of power (Glattfelder and Battiston, 2009; Glattfelder, 2010; Vitali et al., 2011). Understanding how this network of ownership relates to hierarchical power is an important task for future research. It suggests that owners of investment firms may have far greater hierarchical power than the (small) size of their firm indicates.

Third, we want to know if our results extend *beyond* the United States. At present, we do not know. But the methods used here could easily be applied to other countries. This would involve testing if hierarchical power relates to income and class among non-American CEOs. We could then use the hierarchy model to test if CEO trends extend to the general public (in each country). If they do, we would use the model to infer the relation between income, class, and hierarchical power.

Fourth, we want to know if the relation between income, class and hierarchical power has *changed* over time. Again, we have no direct evidence. But there is *indirect* evidence that the recent growth of US top incomes is due to a redistribution of income within firm hierarchies. In a landmark study, Song et al. (2016) find that the recent growth of US top incomes is due mostly to increasing inequality *within* firms. Using a similar model as here, I have found that the growth of top incomes may be due to a redistribution of income towards the tops of firm hierarchies (Fix, 2018b).

Fifth, we want to know if a relation between income, class and hierarchical power extends to non-capitalist societies. I think it should. I have proposed the power-income hypothesis as a universal principle of resource distribution. The rationale is that the desire to use power for personal gain is deeply imprinted on the human psyche. The reasons may be Darwinian. Laura Betzig (1982; 2012; 2018) argues that we (mostly males) seek hierarchical power as a means for achieving reproductive success. While we know little about hierarchy in pre-capitalist societies, we can use a model to project modern trends into the past. Doing so suggests that the growth of hierarchy can possibly explain the origin and evolution of inequality (Fix, 2018a). Again, this is a model-based inference, but it suggests that income plausibly increases with hierarchical power in pre-capitalist societies.

What about the relation between hierarchy and class? Obviously class types change in different societies. But my general hypothesis is that in societies with a mature class system, the income of elites should relate to the ideology of power. Admittedly we have very little data on this topic. But consider how Reinhard Bendix describes the relation between authority, income and property rights in German feudal society:

> ... governmental functions were usable rights which could be sold or leased at will. For example, judicial *authority* was a type of *property*. The person who bought or leased that property was entitled to adjudicate disputes and *receive the fees and penalties* incident to such adjudication. (Bendix, 1980, p. 149)[emphasis added]

If we paraphrase Bendix, we arrive at the same reasoning that I used to derive the capitalist gradient hypothesis. Building on the work of Nitzan and Bichler (2009), I suggested that 'capitalist authority' is a 'type of property'. The person who buys this property is 'entitled' to wield hierarchical power and 'receive income' in return. This reasoning led to hypothesis that the capitalist portion of income should increase with hierarchical power. Of course, Bendix's description hardly means that class is a function of hierarchical power in feudal societies. But it suggests that class-based income in modern societies might not be as different from feudal societies as we would like to think.

To summarize, the study of how hierarchical power relates to income and class is in its infancy. Many questions remain un-investigated and unanswered. But the results here are promising. It seems plausible that, at least in the United States, hierarchical power can help unify the study of income distribution.

7 Conclusions

This paper is proposes an alternative to orthodox theories of income distribution. For more than a century, neoclassical and Marxist theories have been the dominant ways of explaining income. Both assume that value is *produced*. Unfortunately, accepting this idea makes it difficult to explain the skewed distribution of personal income. As long as neoclassical and Marxist ideas remain entrenched, I believe the study of income distribution will stagnate. To echo Nitzan and Bichler (2012), we need a "radical Ctrl-Alt-Del". The goal of this paper has been to chart a new course for studying income and class.

Like any complex phenomenon, income size and income class probably have many causes. But if we want to focus on only *one* correlate, I have proposed it should be *hierarchical power* — the control over subordinates within a hierarchy. This, I have argued, is the key factor that separates the rich from the poor. The rich own and command hierarchies. The poor do not. To connect income and class to hierarchical power, I have proposed two related hypotheses. First, individual income increases with hierarchical power (the power-income hypothesis, Section 2.3). Second, the capitalist *portion* of income increases with hierarchical power (the capitalist gradient hypothesis, Section 2.4).

Evidence from US CEOs supports these hypotheses (Section 4). Among CEOs, income increases with hierarchical power, as does the capitalist portion of this income. Since CEOs are a unique group, I used a model to test if CEO trends extend to the general public (Section 5). While preliminary, the results of this test suggest that there is a general, three-way relation between income size, income class and hierarchical power among the US public. I then used the model to infer what this relation looks like (Section 6). Behind the income and class of the very rich, the model suggests, lies immense hierarchical power.

Because it is an opening salvo, the research here uses sparse and often non-ideal data. But while preliminary, the evidence suggests that hierarchical power provides a plausible basis for unifying the study of income class. The task for the future is to subject this idea to more rigorous testing.

Acknowledgments

Thank-you to Jonathan Nitzan for comments on this paper. Thank-you also to Deepankar Basu, Tim DiMuzio, and Mehrene Larudee for their helpful peer review.

Appendix

Supplementary materials for this paper are available at the Open Science Framework:

https://osf.io/wp8yu/

The supplementary materials include:

1. Source data;
2. Code for all analysis;
3. Hierarchy model code.

A US Class-Based Income

Data for US class-based income comes from the World Inequality Database (WID). I use this data in Figures 10–13. My income measures are shown in Table 8. These are composed of the WID data series shown in Tables 9 and 10. I use two WID series to construct K1*, L1*, and T. This means I merge statistics from both WID series.

The WID data comes from Piketty et al. (2017b). For the methods of this study, see Piketty et al. (2017a). This is the most detailed study to date of US class-based income. However, it comes with some caveats. Piketty et al. subdivide proprietor income into capitalist and labor components. The capitalist component is series *fkbus*. The labor component is series *flmil* (Table 10). Oddly, I cannot find an explicit statement of the methods behind this split in Piketty's work. According to Rognlie (2016), Piketty assumes that proprietor income "has the same net capital share as the corporate sector".

This leads to a difference between my definitions of class-based income (Table 1) and the empirical data (Table 8). My two-class definition of capitalist income (K1) includes *all* proprietor income. In contrast, the empirical measure K1* contains only a portion of proprietor income. My definition of labor income (L1) contains *no* proprietor income. In contrast, the empirical measure L1* contains a portion of proprietor income.

In addition to the capitalist income series provided by WID, I construct my own series K2 shown in Table 8. This includes equity and interest income (with capital gains). It corresponds to capitaist income K2, defined in Table 2.

Methods for Estimating Income Distribution Statistics

WID provides three types of data that I use to compute statistics:

1. Income percentile (bin)
2. Income share (by income percentile bin)
3. Income threshold (by income percentile bin)

As an example, the WID data may indicate that percentiles P99–P100 have an income share of 15%. This means the top 1% holds 15% of all income. The income threshold for this bin may be $200,000. This means that the lowest income of the top 1% is $200,000.

Table 8: Measures of US Class-Based Income

Measure	Symbol	Composition
Capitalist Income (All Ownership)	K1*	both fkinc and pkinc
Capitalist Income (Scalable Ownership Only)	K2	fkequ + fkfix
Labor Income	L1*	both flinc and plinc
Total Income	T	both fainc and ptinc

Table 9: World Inequality Database Main Series

Series	Description	Composition
fainc	Personal factor income	flinc + fkinc
fkinc	Personal factor capital income	fkhou + fkequ + fkfix + fkbus + fkpen + fkdeb
flinc	Personal factor labor income	flemp + flmil + flprl
pkinc	Personal pre-tax capital income	fkinc + pkpen + pkbek
plinc	Personal pre-tax labor income	flinc + plcon + plbel
ptinc	Personal pre-tax income	plinc + pkinc

Table 10: World Inequality Database Component Series

Series	Description
fkbus	Business asset income
fkdeb	Interest payments
fkequ	Equity asset income
fkfix	Interest income
fkhou	Housing asset income
fkpen	Pension and insurance asset income
flemp	Compensation of employees
flmil	Labor share of net mixed income
flprl	Sales and excise taxes falling on labor
pkbek	Capital share of social insurance income
pkpen	(Minus) Investment income payable to pension funds
plbel	Labor share of social insurance income
plcon	(Minus) social contributions

Gini Index: I estimate the Gini index by constructing a Lorenz curve from WID data. The Gini index equals the area between the Lorenz curve and the line of perfect equality, divided by the total area under the line of perfect equality.

Top 1% Share: This is provided directly by the WID data.

Power-Law Exponent: I estimate the power-law exponent of the top 1% of incomes using income percentile and threshold data. I create binned data where we know the proportion of people in each bin, and the income boundaries of each bin. I then use the method discussed in Virkar and Clauset (2014) to estimate the power-law exponent from this binned data.

Probability Density: I estimate the probability density using income percentile and threshold data. I first normalize income threshold data so that the median equals 1. I then construct a cumulative distribution. This is the fraction of individuals below a given income. I estimate the probability density function from the *slope* of the cumulative distribution.

Lorenz Curve: The Lorenz curve is constructed from income percentile and income share data. It is the cumulative share of income vs. income percentile.

Cumulative Distribution: I construct the cumulative distribution from income percentile and threshold data. I normalize income threshold data so that the median equals 1.

Complementary Cumulative Distribution: I construct the complementary cumulative distribution (CCD) from the cumulative distribution (CD). The y-value for the CCD is 1 minus the corresponding y-value for the CD.

Capitalist Income Share vs. Percentile: Capitalist income share $\mathrm{K1}^*_{\mathrm{frac}}$ is calculated by merging two series:

$$\mathrm{K1}^*_{\mathrm{frac}} = \begin{cases} \text{fkinc / fainc} \\ \text{pkinc / ptinc} \end{cases} \tag{8}$$

Capitalist income share $K2^{*}_{\text{frac}}$ is calculated as:

$$K2^{*}_{\text{frac}} = (\text{fkequ} + \text{fkfix}) \;/\; \text{fainc} \tag{9}$$

B Case-Study Firms

I review here the evidence from firm case studies that informs the hierarchy model. Table 11 summarizes the source data, while Figure 15 shows the hierarchical employment and pay structure of these firms. The firms remain anonymous, and are named after the authors of the case-study papers. Although the exact shapes vary, the employment structure of each firm has a rough *pyramid* shape. The pay structure of each firm has an *inverse* pyramid shape.

Figure 16 dissects these trends. Figure 16A shows how the span of control (the employment ratio between adjacent ranks) changes as a function of hierarchical level. In these firms, the span of control is not constant, but instead tends to *increase* with hierarchical rank. Similarly, Figure 16B shows the ratio of mean pay between adjacent ranks. Like the span of control, the pay ratio tends to increase with hierarchical rank. Lastly, Figure 16C shows income dispersion within hierarchical ranks of each firm (measured with the Gini index). Note that income dispersion within ranks is quite low and there is no evidence of a trend.

The case-study data plays a central role in the hierarchical model developed in this paper. From the case-study evidence, I propose the following 'stylized' facts:

1. The span of control tends to *increase* with hierarchical level.
2. The inter-level pay ratio tends to *increase* with hierarchical level.
3. Intra-level income inequality is approximately *constant* across all hierarchical levels.

The case-study evidence informs the basic structure of the model, and also some of its key parameters. The 'shape' of modeled firm hierarchies is determined from the fitted span-of-control trend shown in Figure 16A. Figure 17 shows the idealized employment hierarchy that is implied by case-study data. Error bars indicate uncertainty, calculated using the bootstrap resampling method. Parameters for income dispersion within ranks are determined from the mean of data in Figure 16C. For a detailed discussion of the model algorithm and parameter-fitting procedure, see Sections D and E.

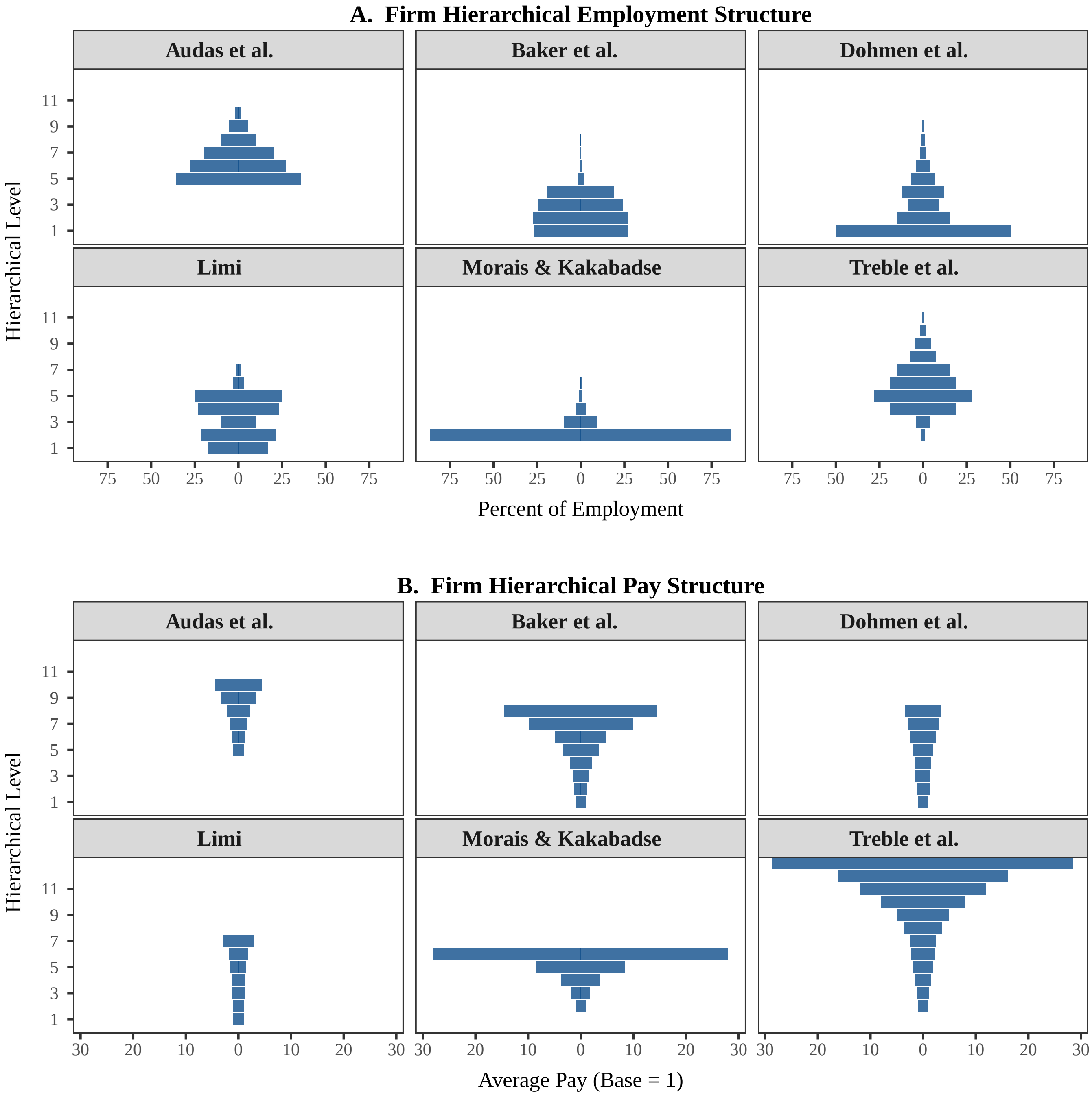

Figure 15: The Hierarchical 'Shape' of Six Different Case-Study Firms

This figure shows the hierarchical employment and pay structure of six different case-study firms. Panel A shows the hierarchical structure of employment, while Panel B shows the hierarchical pay structure.

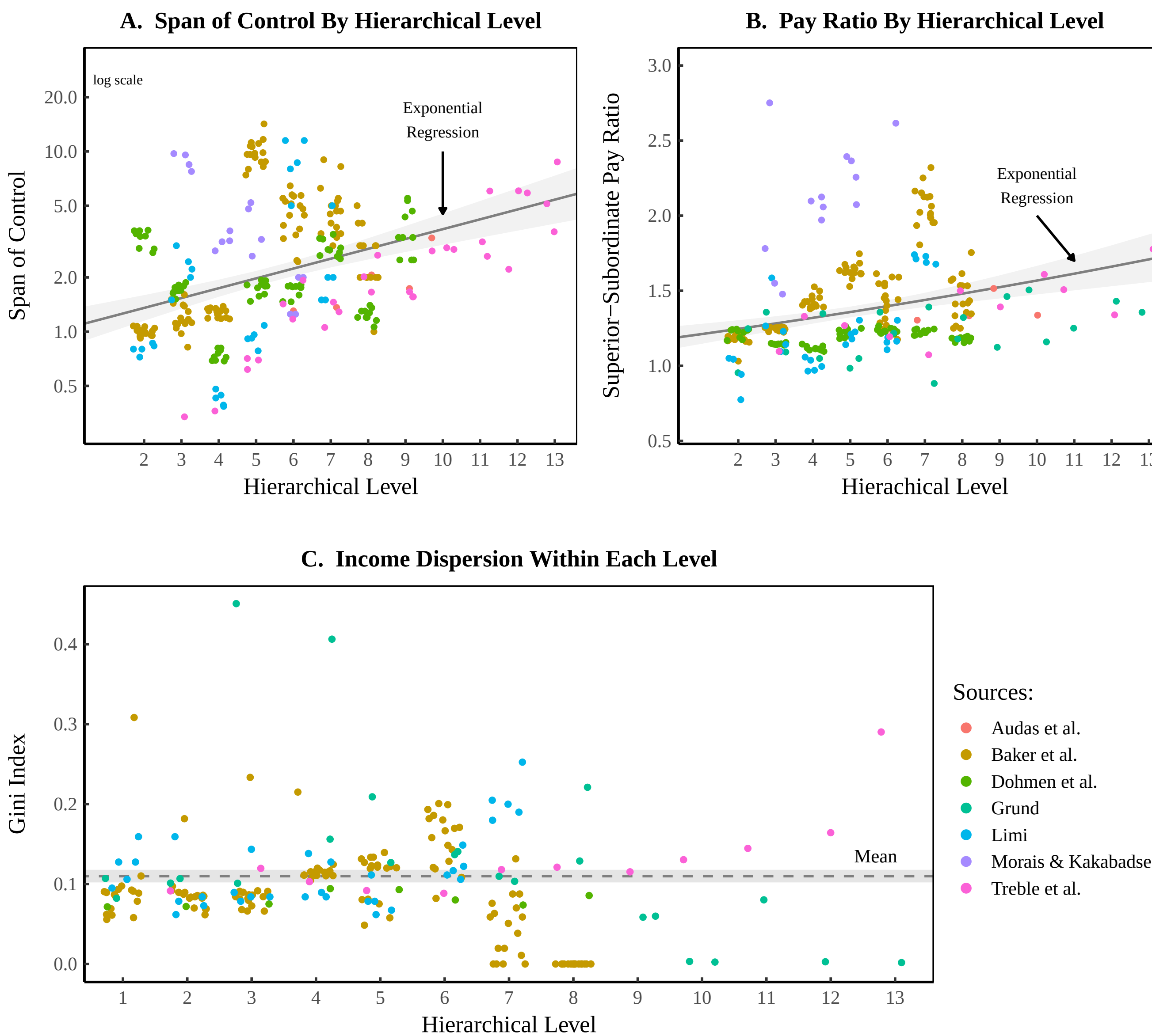

Figure 16: Analyzing the Hierarchical Structure of Case-Study Firms

This figure shows data from 7 case-study firms. Note that Grund (2005) appears in this figure, but not in Figures 15. This is because Grund does not provide data for employment by hierarchical rank (see Tbl. 11). Panel A shows how the span of control (the subordinate-to-superior employment ratio between adjacent levels) varies with hierarchical level. Note the log scale on the y-axis. Panel B shows how the superior-to-subordinate pay ratio varies with hierarchical level. In Panels A and B, the *x*-axis corresponds to the *upper* hierarchical level in each corresponding ratio. Panel C shows the Gini index of income inequality within each hierarchical level. Different case-study firms are indicated by color, with names indicating the study author. Note that horizontal 'jitter' has been introduced in all three plots in order to better visualize the data (hierarchical level is a discrete variable). The lines in Panels A and B indicate exponential regressions, while the line in Panel C shows the average Gini index. Grey regions correspond to the 95% confidence intervals.

Table 11: Summary of Firm Case Studies

Source	Years	Country	Firm Levels	Span of Control	Level Income	Level Income Dispersion
Audas et al. (2004)	1992	Britain	All	✓	✓	
Baker et al. (1993)	1969-1985	United States	Management	✓	✓	✓
Dohmen et al. (2004)	1987-1996	Netherlands	All	✓	✓	✓
Grund (2005)	1995 & 1998	US and Germany	All		✓	✓
Lima (2000)	1991-1995	Portugal	All	✓	✓	✓
Morais and Kakabadse (2014)*	2007-2010	Undisclosed	All	✓	✓	
Treble et al. (2001)	1989-1994	Britain	All	✓	✓	✓

Notes: This table shows metadata for the firm case studies displayed in Fig. 16. The 'Firm Levels' column refers to the portion of the firm that is included in the study. 'Management' indicates that only management levels were studied.
*I discard (as an outlier) the bottom hierarchical level in Morais and Kakabadse's data.

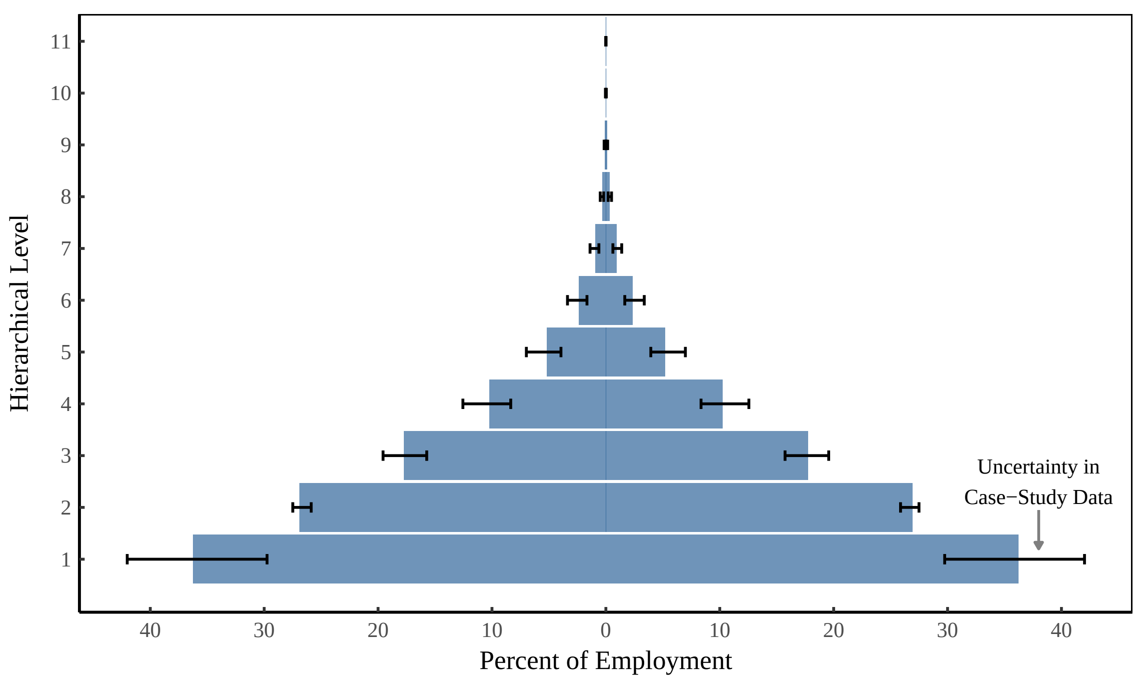

Figure 17: Idealized Firm Employment Hierarchy Implied by Case Studies

This figure shows the idealized firm hierarchy that is implied by fitting trends to case-study data (Fig. 16A). Error bars show the uncertainty in the hierarchical shape, calculated using a bootstrap resample of case-study data.

C US CEO Data

Data for US CEOs (and their firms) comes from the Execucomp and Compustat databases. I use this data for the case study of CEO pay (Section 4) and as the basis for the US hierarchy model (Section 5). Methods are discussed below.

Finding the CEO

I identify CEOs using titles in the Execucomp series TITLEANN. I use a three-step algorithm:

1. Find all executives whose title contains one or more of the words in the 'CEO Titles' list in Table 12.
2. Of these executives, take the subset whose title does *not* contain any of the words in the 'Subordinate Titles' list in Table 12.
3. If this returns more than one executive per firm per year, chose the executive with the highest pay.

Table 12: Titles Used to Identify the 'CEO'

CEO Titles:	Subordinate Titles
president	vp
chairman	v-p
CEO	cfo
Chief Executive Officer	vice
chmn	chief finance officer
	president of
	coo
	division
	div
	president-
	group president
	chairmain-
	co-president
	deputy chairman
	pres.-
	Chief Financial Officer

Titles such as 'president-' and 'president of' are included in the subordinate list because they typically refer to a president of a division within the company: i.e. 'president of western division' or 'president-western hemisphere'.

Table 13: Execucomp Compensation Series

Series	Description
TOTAL_ALT2	SALARY + BONUS + OTHCOMP + NONEQ_INCENT + PENSION_CHG + OPT_EXER_VAL + SHRS_VEST_VAL
BONUS	The dollar value of a bonus earned by the named executive officer during the fiscal year.
SALARY	The dollar value of the base salary earned by the named executive officer during the fiscal year.
OTHCOMP	Other compensation received by the director including perquisites and other personal benefits, contributions to defined contribution plans (e.g. 401K plans), life insurance premiums, gross-ups and other tax reimbursements, discounted share purchases, consulting fees, awards under charitable award programs, etc.
NONEQ_INCENT	Value of amounts earned during the year pursuant to non-equity incentive plans.
PENSION_CHG	Composed of a) above-market or preferential earnings from deferred compensation plans, and b) aggregate increase in actual value of defined benefit and actual pension plans during the year.
OPT_EXER_VAL	Value realized from option exercises during the year. The value is calculated as of the date of exercise and is based on the difference between the exercise price and the market price of the stock on the exercise date.
SHRS_VEST_VAL	Value of restricted shares that vested during the year.

CEO Pay and Capitalist Income Fraction

Execucomp contains several different estimates of CEO pay. These differ primarily in the valuation of stock option compensation. Hopkins and Lazonick (2016) argue that we should use *actual realized gains*. This is the difference between the *market value* of the option and the *exercise value* at the time of exercise. Importantly, actual realized gains is the income recorded by the IRS for tax purposes. I measure CEO total pay and capitalist income fraction (K_{frac}) using the following series:

$$\text{Total Pay} = \text{TOTAL\_ALT2} \tag{10}$$

$$K_{\text{frac}} = \frac{\text{Actual Realized Gains from Stock Options}}{\text{Total Pay}} \tag{11}$$

$$K_{\text{frac}} = \frac{\text{SHRS\_VEST\_VAL} + \text{OPT\_EXER\_VAL}}{\text{TOTAL\_ALT2}} \quad (12)$$

Series descriptions are shown in Table 13.

CEO Pay Ratio and Firm Employment

I calculate the CEO pay ratio as:

$$\text{CEO Pay Ratio} = \frac{\text{CEO Pay}}{\text{Firm Mean Income}} \quad (13)$$

Firm mean income is calculated by dividing total staff expenses (Compustat Series XLR) by total employment (Compustat Series EMP):

$$\text{Firm Mean Income} = \frac{\text{Total Staff Expenses}}{\text{Total Employment}} \quad (14)$$

CEO pay ratio and firm mean income data are available for roughly 3000 firm-year observations from 2006-2016. Figure 18 shows summary statistics of this data.

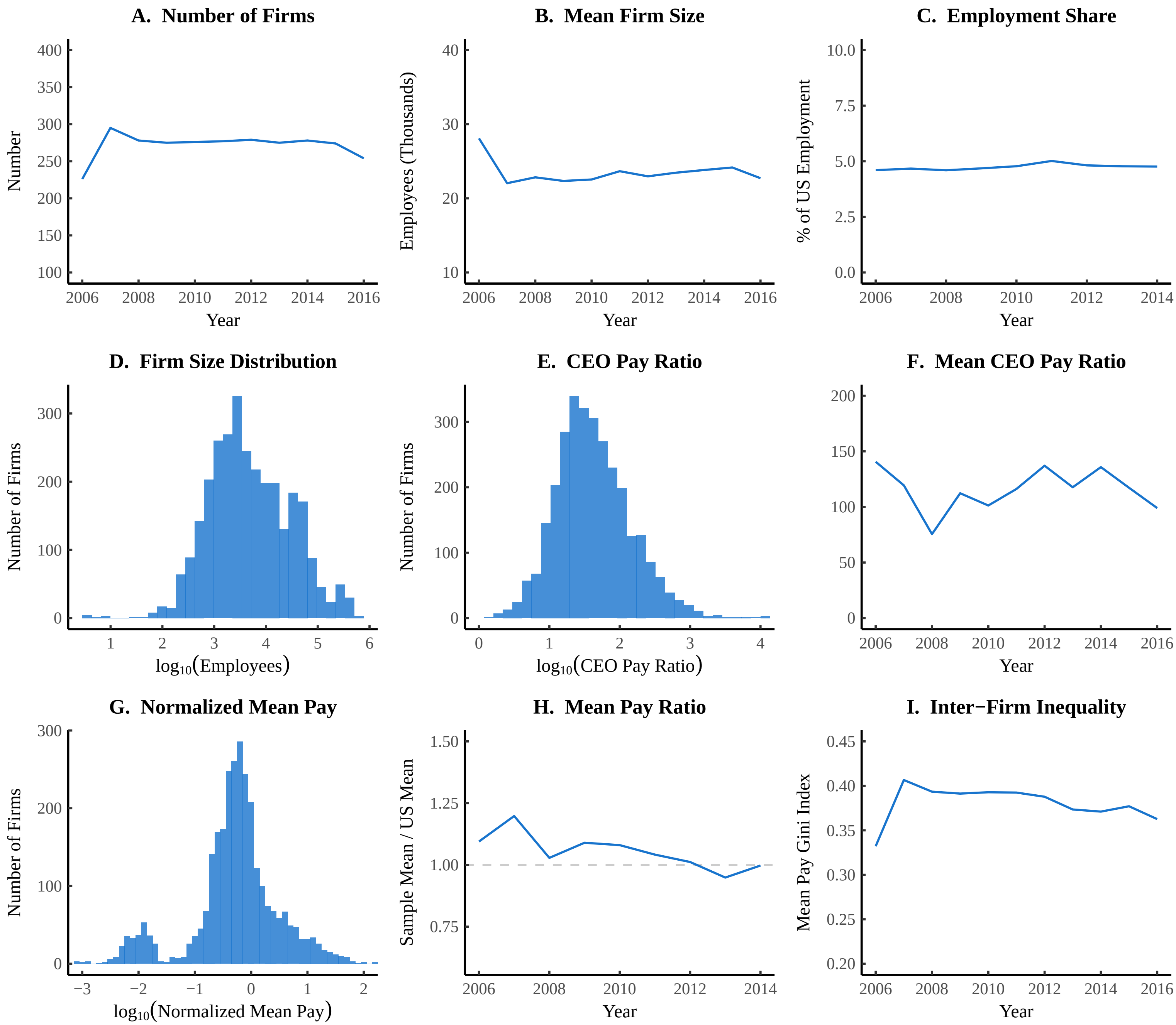

Figure 18: Statistics of the CEO Firm Sample

This figure shows selected statistics of the CEO firm sample. Panel A shows the number of firms in the sample over time, Panel B the average firm size, and Panel C the share of US employment held by these firms. Panel D shows the logarithmic distribution of firm size, and Panel E shows the logarithmic distribution of the CEO pay ratio. Panel F shows the mean CEO pay ratio of all firms over time. Panel G shows the logarithmic distribution of normalized mean pay (mean pay divided by the average pay of the firm sample in each year). Panel H shows the ratio of mean pay in the sample relative to the US average (calculated from BEA Table 1.12 by dividing the sum of employee and proprietor income by the number of workers in BEA Table 6.8C-D. Panel I shows the Gini index of firm mean pay over time.

D Hierarchy Model Equations

I discuss here the equations of the hierarchy model used in Section D. Based on evidence from firm case studies (Section B), the model assumes the following:

1. Firms are hierarchically structured, with a span of control that increases *exponentially* with hierarchical level.
2. The ratio of mean pay between adjacent hierarchical levels increases *exponentially* with hierarchical level.
3. Income within hierarchical levels is lognormally distributed and does not vary between levels.

I use these assumptions to create equations describing the hierarchical employment and pay structure of firms. For notation, see Table 14.

Table 14: Notation

Symbol	Definition
a	span of control parameter 1
b	span of control parameter 2
C	CEO to average employee pay ratio
E	employment
F	cumulative distribution function
G	Gini index of inequality
h	hierarchical level
$\bar{I}$	average income
μ	lognormal location parameter
n	number of hierarchical levels in a firm
p	pay ratio between adjacent hierarchical levels
r	pay-scaling parameter
s	span of control
σ	lognormal scale parameter
T	total for firm
$\downarrow$	round down to nearest integer
$\prod$	product of a sequence of numbers
$\sum$	sum of a sequence of numbers

D.1 Generating the Employment Hierarchy

To generate the employment hierarchy of a firm, we begin by defining the *span of control* (s). It is the average number of direct subordinates controlled by a given individual. In the model, we define this as the ratio of employment (E) between two consecutive hierarchical levels (h). We let $h = 1$ be the *bottom* hierarchical level (bottom-ranked individuals). We define the span of control in level 1 as $s = 1$. This leads to the following piecewise function:

$$s_h \equiv \begin{cases} 1 & \text{if } \; h = 1 \\ \dfrac{E_{h-1}}{E_h} & \text{if } \; h \geq 2 \end{cases} \tag{15}$$

Based on the firm case studies (Section B), we assume that the span of control increases *exponentially* with hierarchical level, with a and b as free parameters:

$$s_h = \begin{cases} 1 & \text{if } \; h = 1 \\ a \cdot e^{bh} & \text{if } \; h \geq 2 \end{cases} \tag{16}$$

As one moves up the hierarchy, employment in each consecutive level (E_h) *decreases* by $1/s_h$. This yields Eq. 17, a recursive method for calculating E_h. Since we want employment to be *whole* numbers, we round down to the nearest integer (notated by $\downarrow$).

$$E_h =\downarrow \frac{E_{h-1}}{s_h} \qquad \text{for} \qquad h > 1 \tag{17}$$

By repeatedly substituting Eq. 17 into itself, we can obtain a non-recursive formula:

$$E_h =\downarrow E_1 \cdot \frac{1}{s_2} \cdot \frac{1}{s_3} \cdot \ldots \cdot \frac{1}{s_h} \tag{18}$$

In product notation, Eq. 18 can be written as:

$$E_h =\downarrow E_1 \prod_{i=1}^{h} \frac{1}{s_i} \tag{19}$$

Total employment in the whole firm (E_T) is the sum of employment in all hierarchical levels. Defining n as the total number of hierarchical levels, we get Eq. 20, which in summation notation, becomes Eq. 21.

$$E_T = E_1 + E_2 + \ldots + E_n \tag{20}$$

$$E_T = \sum_{h=1}^{n} E_h \tag{21}$$

The model 'builds' the hierarchy from the bottom up. Therefore, n is not known beforehand, so we define it using Eq. 19. We progressively increase h until we reach a level of zero employment. The highest level n will be the hierarchical level directly *below* the first hierarchical level with zero employment:

$$n = \{h \quad | \quad E_h \geq 1 \quad \text{and} \quad E_{h+1} = 0\} \tag{22}$$

To summarize, the employment hierarchy of our model firm is determined by 3 free parameters: the span of control parameters a and b, and base-level employment E_1. Code for this algorithm is located in `exponents.h` in the Supplementary Material.

D.2 Generating Hierarchical Pay

To model the hierarchical pay, we begin by defining the inter-hierarchical pay-ratio (p_h). It is the ratio of mean income ($\bar{I}$) between adjacent hierarchical levels. Again, it is helpful to use a piecewise function so that we can define a pay-ratio for hierarchical level 1:

$$p_h \equiv \begin{cases} 1 & \text{if } h = 1 \\ \dfrac{\bar{I}_h}{\bar{I}_{h-1}} & \text{if } h \geq 2 \end{cases} \tag{23}$$

Based on the case-study evidence (Section B), we assume that the pay ratio increases *exponentially* with hierarchical level. I model this with the following function, where r is a free parameter:

$$p_h = \begin{cases} 1 & \text{if } h = 1 \\ r^h & \text{if } h \geq 2 \end{cases} \tag{24}$$

Using the same logic as with the employment hierarchy, the mean income (I_h) in any hierarchical level is defined recursively by Eq. 25 and non-recursively by Eq. 26.

$$\bar{I}_h = \frac{\bar{I}_{h-1}}{p_h} \tag{25}$$

$$\bar{I}_h = \bar{I}_1 \prod_{i=1}^{h} p_i \tag{26}$$

To summarize, the pay hierarchy of our model firm is determined by 2 free parameters: the pay-scaling parameter r, and mean pay in the base level ($\bar{I}_1$). Code for this algorithm is located in `model.h` in the Supplementary Material.

D.2.1 Useful Statistics

Two statistics are used repeatedly within the model: mean pay within the firm, and the CEO-to-average-employee pay ratio.

Mean income for all employees ($\bar{I}_T$) is equal to the weighted average of income by rank. It is the average of mean income in each hierarchical level ($\bar{I}_h$), weighted by the respective hierarchical level employment (E_h):

$$\bar{I}_T = \sum_{h=1}^{n} \bar{I}_h \cdot \frac{E_h}{E_T} \tag{27}$$

To calculate the CEO pay ratio, we define the CEO as the person(s) in the top hierarchical level. Therefore, CEO pay is simply $\bar{I}_n$, average income in the top hierarchical level. The CEO pay ratio (C) is then equal to CEO pay divided by average pay:

$$C = \frac{\bar{I}_n}{\bar{I}_T} \tag{28}$$

D.3 Adding Intra-Level Pay Dispersion

Up to this point, we have modeled only the *mean* income within each hierarchical level of a firm. The last step is to add pay *dispersion* within each hierarchical level.

I assume that pay dispersion within hierarchical levels is *lognormally* distributed. The lognormal distribution is defined by location parameter μ and scale parameter σ. The case-study evidence (Section B) suggests that pay dispersion within hierarchical levels is relatively constant (see Fig. 16C). Thus, the model assumes *identical inequality* within all hierarchical levels. This means that the lognormal scale parameter σ is the same for all hierarchical levels.

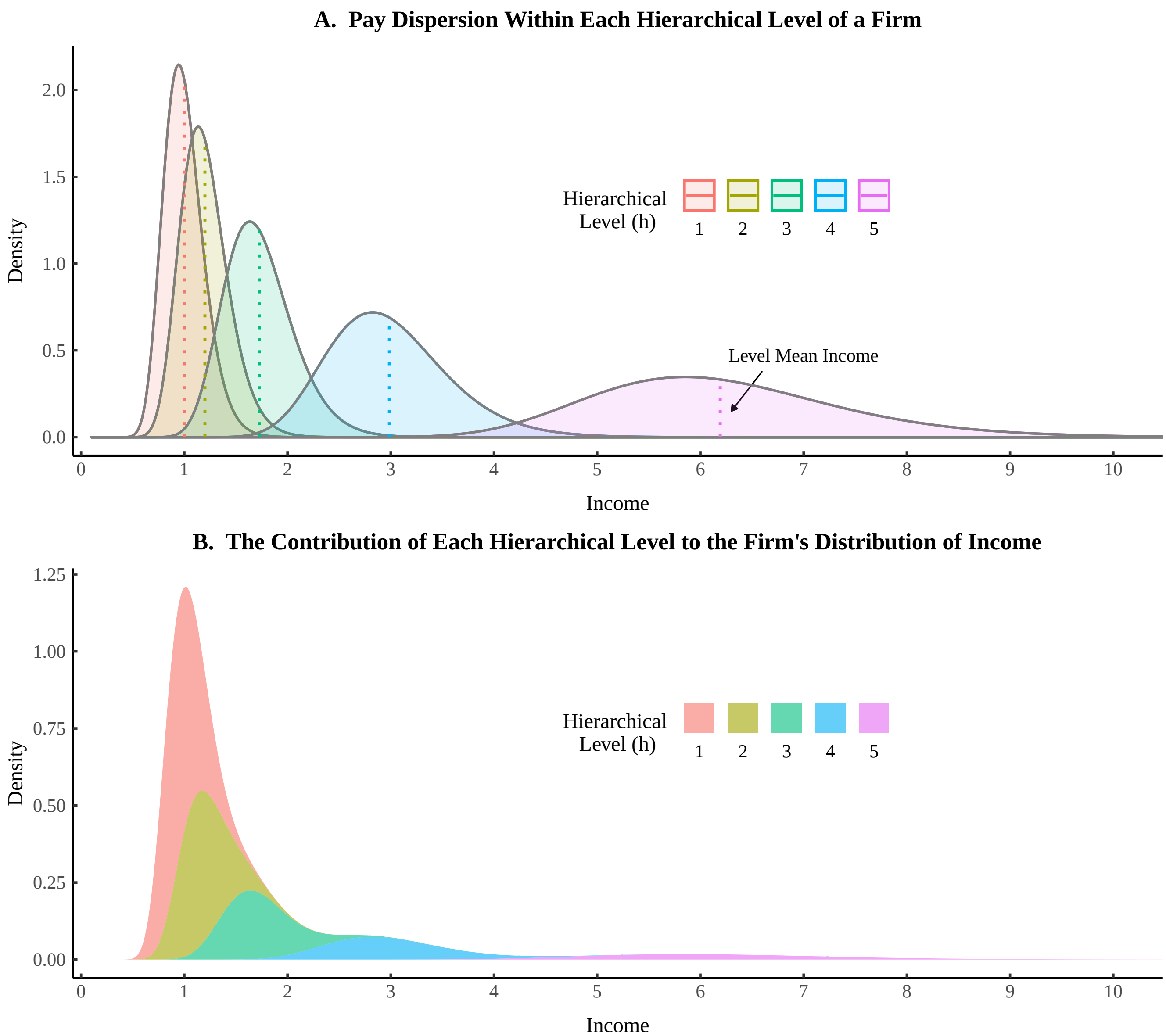

Figure 19: Adding Intra-Level Pay Dispersion to a Model Firm

This figure illustrates pay dispersion within a model firm. Colors show pay dispersion within each hierarchical level. Panel A shows the income distributions for each level, with means indicated by a dashed vertical line. Panel B shows the contribution of each hierarchical level to the income distribution of the whole firm. Income density functions are summed by weighting their respective employment. This firm has a pay-scaling parameter of $r = 1.2$ and an intra-level Gini index of 0.13.

To add dispersion within each hierarchical level, I multiply mean pay $\bar{I}_h$ by a lognormal random variate with an expected mean of one:

$$I_h = \bar{I}_h \cdot \ln \mathcal{N}(\mu, \sigma) \tag{29}$$

The mean of a lognormal distribution is $e^{\mu + \frac{1}{2}\sigma^2}$. For the mean to be 1, μ must be:

$$\mu = -\frac{1}{2}\sigma^2 \tag{30}$$

Given a value for σ (a free parameter), we can define the pay distribution within any hierarchical level of a firm. This process is shown graphically in Figure 19. Figure 19A shows the income distributions for each hierarchical level of a 5-level firm. Figure 19B shows the contribution of each hierarchical level to the firm's income distribution. Lower levels have more members and thus dominate the distribution. Code for this algorithm is in `model.h` in the Supplementary Material.

Calculating Hierarchical Power in the Hierarchy Model

I define an individual's hierarchical power (P) as the number of subordinates (S) under their control, plus 1:

$$P = S + 1 \tag{31}$$

Because the hierarchy model simulates only the *aggregate* structure of firms (employment by hierarchical level), hierarchical power is calculated as an *average* per rank. For hierarchical rank h, the average hierarchical power ($\bar{P}_h$) is defined as the average number of subordinates ($\bar{S}_h$) plus 1:

$$\bar{P}_h = \bar{S}_h + 1 \tag{32}$$

Within each rank, every individual is assigned the average power $\bar{P}_h$. The average number of subordinates $\bar{S}_h$ is equal to the sum of employment (E) in all subordinate levels, divided by employment in the level in question:

$$\bar{S}_h = \sum_{i=1}^{h-1} \frac{E_i}{E_h} \tag{33}$$

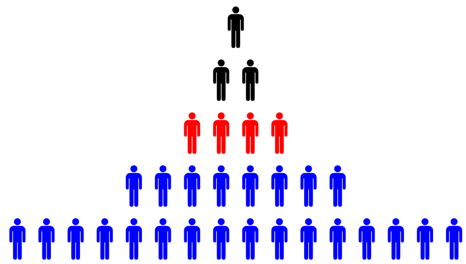

Figure 20: Calculating the Average Number of Subordinates

As an example, consider the hierarchy in Figure 20. The average number of subordinates below each individual in hierarchical level 3 (red) is:

$$\bar{S}_3 = \frac{E_1 + E_2}{E_3} = \frac{16+8}{4} = 6 \tag{34}$$

Therefore, these individuals would all be assigned a hierarchical power of 7.

E The United States Hierarchy Model

I review here the technical details of the US hierarchy model. The model uses the hierarchy algorithms from Section D and applies them to the United States. The model's parameters are summarized in Table 15. The technical structure of the model is summarized in Table 16.

The model has three main steps. First, it uses firm case-study data and CEO pay-ratio data to simulate the hierarchical structure within CEOs' firms. Second, the model generalizes this simulation to a firm size distribution that is representative of the US. Third, the model assigns a class component to individual income.

E.1 Step 1. Simulate the Hierarchical Structure of CEOs' Firms

The first step of the hierarchy model is to simulate the hierarchical structure of our sample of CEOs' firms (Section C). To build the employment hierarchy, the model assumes CEOs' firms have the same hierarchical 'shape' as firm case

Table 15: Parameters in the US Hierarchy Model

Parameter	Definition	Action	Scope
α	Power-law exponent for the size distribution of firms	Determines the skewness of the firm size distribution	—
a, b	Span of control parameters	Determines the shape of the firm hierarchy.	Identical for all firms.
E_1	Employment in base hierarchical level	Used to build the employment hierarchy from the bottom up. Determines total employment.	Specific to each firm.
r	Pay-scaling parameter	Determines the rate at which mean income (within a firm) increases by hierarchical level.	Specific to each firm.
$\bar{I}_h$	Mean pay in base hierarchical level	Sets the base level income of the firm, which determines firm average pay.	Specific to each firm.
σ	Within-rank pay dispersion parameter	Determines the level of inequality within hierarchical ranks of a firm.	Identical for all firms.
κ	Capitalist gradient parameter	Determines how rapidly the capitalist fraction of income increases with hierarchical power	Identical for all individuals.

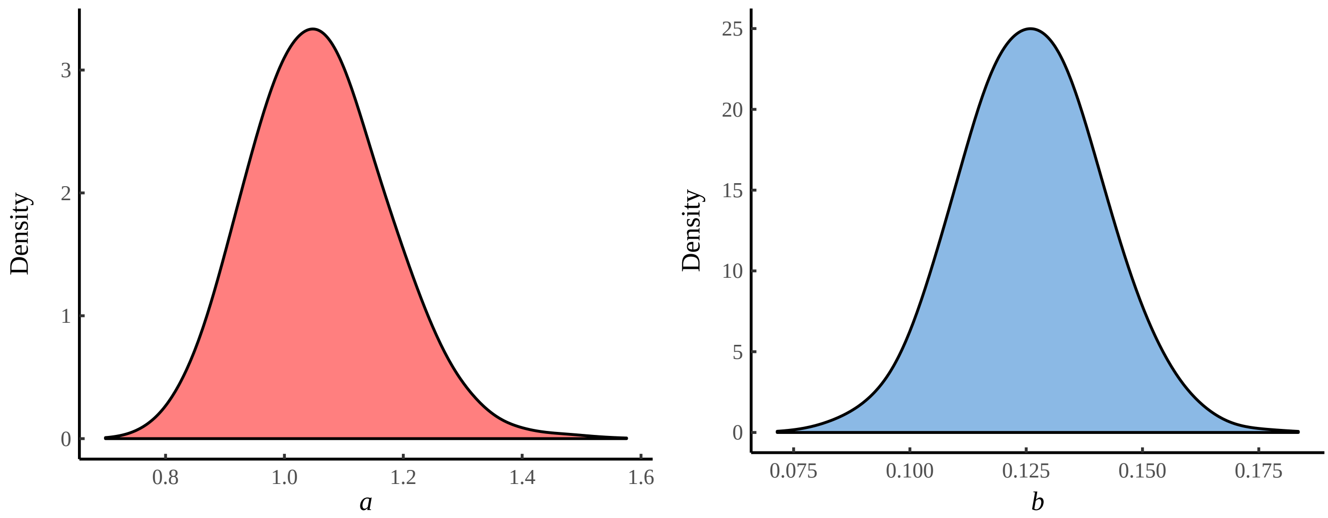

Figure 21: Uncertainty in the Span of Control Parameters

This figure shows the uncertainty in the span-of-control parameters a and b. Together, these parameters determine the 'shape' of the firm hierarchy. These parameters are determined from regressions on firm case-study data (Fig. 16). I estimate uncertainty using the bootstrap technique.

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studies (Section B). The model then uses the empirical CEO pay ratio to infer the pay hierarchy within each CEO firm.

E.1.1 The Employment Hierarchy of CEOs' Firms

The model assumes that all CEO firms have the same hierarchical shape as the average shape found in firm case studies (Fig. 17). This shape is determined by the span of control parameters a and b, which are estimated from an exponential regression on case-study data (Fig. 16A). The model assumes these parameters are *constant* for all firms, meaning all firms have the same hierarchical shape.

Because there are only six firms in the case-study sample, there is significant uncertainty in the values of a and b. I incorporate this uncertainty into the model using the *bootstrap* method (Efron and Tibshirani, 1994). I repeatedly resample the case-study data (with replacement) and estimate the parameters a and b for each resample. Figure 21 shows the resulting uncertainty in a and b. I include this uncertainty in the model by using a different resample of a and b in each iteration. Code implementing this bootstrap is located in `boot_span.h` in the Supplementary Material.

After receiving parameters a and b from a bootstrap sample, the model con-

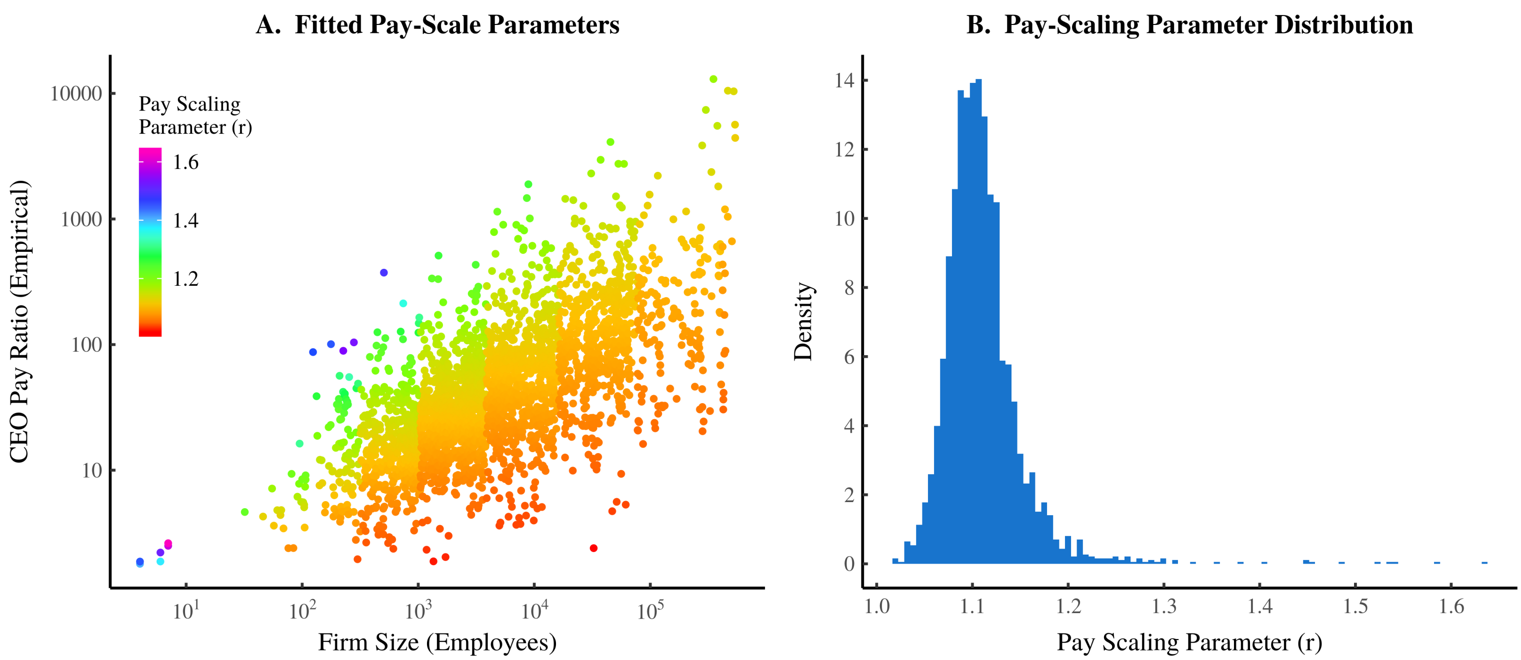

Figure 22: Fitting CEOs' Firms with a Pay-Scaling Parameter

This figure shows the fitted pay-scaling parameters (r) for all CEOs' firms. This parameter determines how rapidly income increases with hierarchical rank. Panel A shows the relation between the CEO pay ratio, firm size, and r. The discrete changes in color (evident as vertical lines) correspond to changes in the number of hierarchical levels within firms. The distribution of r for all firms (and years) is shown in panel B.

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structs the employment hierarchy of each CEO firm. The input empirical data is the firm's total employment. The model constructs the employment hierarchy from the bottom up. To do this, it needs to first estimate employment in the base level of each CEO firm. I use the model to reverse engineer this calculation. I input a range of different base-employment values into equations 16, 19, and 21 and calculate total employment for each value. The result is a discrete mapping relating base employment to total employment. I then interpolate between these values to create a continuous function that predicts base level E_1, given total employment E_T. I use this equation to infer base-level employment in each CEO firm. Code implementing this method is located in `base_fit.h` in the Supplementary Material.

Once we have base-level employment in each CEO firm, the model uses Eqs. 16, 19, and 21 to simulate the employment hierarchy of each CEO firm.

E.1.2 The Pay Hierarchy in CEOs' Firms

The model's next step is to infer the pay hierarchy for each CEO firm. Unlike the employment hierarchy, the model allows the pay hierarchy to *vary* between firms. We assume that hierarchical pay obeys Eqs. 24 and 26, meaning it is determined by the pay-scaling parameter r.

Once the employment hierarchy is set for each CEO firm, r can be estimated from the empirical CEO pay ratio. To solve for r, I use the bisection method to minimize the following error function:

$$\epsilon(r) = \left| \, C_{\text{model}} - C_{\text{empirical}} \right| \tag{35}$$

Here C_{model} and C_{CEO} are modeled and empirical CEO pay ratios, respectively. For each firm, we choose r that minimizes the error function. To ensure that there are no large errors, I discard CEO firms for which the best-fit r parameter gives an error larger than $\epsilon = 0.01$. Figure 22 shows an example of the fitted results for r. Every bootstrap iteration will return slightly different values for r.

Once we have estimated r for each CEO firm, we can estimate pay in the base hierarchical level. To do this, we set up a ratio between base-level pay ($\bar{I}_1$) and firm mean pay ($\bar{I}_T$) for both the model and empirical data:

$$\frac{\bar{I}_1^{\text{ empirical}}}{\bar{I}_T^{\text{empirical}}} = \frac{\bar{I}_1^{\text{ model}}}{\bar{I}_T^{\text{ model}}} \tag{36}$$

The modeled ratio $\bar{I}_1^{\text{ model}} / \bar{I}_T^{\text{ model}}$ is *independent* of the choice of base pay. This is because firm mean pay in the model is a *function* of base pay (see Eq. 26 and 27). If we run the model with $\bar{I}_1^{\text{ model}} = 1$, then Eq. 36 reduces to:

$$\frac{\bar{I}_1^{\text{ empirical}}}{\bar{I}_T^{\text{ empirical}}} = \frac{1}{\bar{I}_T^{\text{ model}}} \tag{37}$$

We then rearrange Eq. 37 to estimate base pay for each CEO firm ($\bar{I}_1^{\text{ empirical}}$):

$$\bar{I}_1^{\text{ empirical}} = \frac{\bar{I}_T^{\text{ empirical}}}{\bar{I}_T^{\text{ model}}} \tag{38}$$

Note that the CEO firm data contains observations over *multiple* years. To adjust for inflation, I divide firm mean pay by the sample average in the given year. Code implementing this fitting method for $\bar{I}_1$ and r is located in `fit_model.h` in the Supplementary Material.

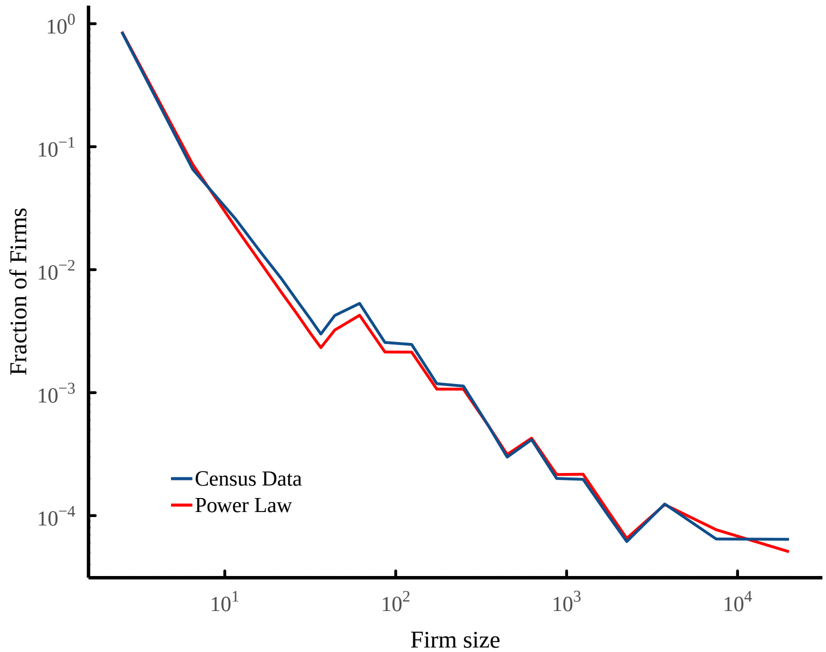

Figure 23: The United States Firm Size Distribution as a Power Law

This figure compares the firm size distribution in the United States to a discrete power-law distribution with exponent $\alpha = 2.01$. The US data combines 'employer' firms and unincorporated self-employed workers. Data for 'employer' firms is from the US Census Bureau, Statistics of U.S. Businesses (using data for 2013). This data is augmented with Bureau of Labor Statistics data for unincorporated self-employed workers (series LNU02032185 and LNU02032192). The histogram preserves firm-size bins used by the Census. I add self-employed individuals to the first bin. The last histogram bin contains all firms with more than 10,000 employees.

E.2 Step 2: Simulate the Whole US Firm Population

In step 2, the model takes the CEO firm simulation and extrapolates it to a firm size distribution that is consistent with the US. Rather than the lognormal distribution found in CEO firms (Fig. 18D), step 2 uses a power-law distribution of firm sizes.

The model generates a power-law sample of firms and then simulates their hierarchical structure. This involves fitting probability distributions to the parameters that were estimated in step 1. The model then uses these parameter distributions to simulate the pay hierarchy within the power-law sample of firms.

E.2.1 Create A Representative Size Distribution of Firms

In the CEO sample, firm size is lognormally distributed (Fig. 18D). Unfortunately, this is not representative of the United States. Recent studies suggest that in the US and other G7 countries, the firm size distribution follows a power law (Axtell, 2001; Gaffeo et al., 2003). In a power-law distribution, the probability of finding a firm of size x is:

$$p(x) \propto \frac{1}{x^{\alpha}} \tag{39}$$

The exponent α determines how skewed the distribution is. Figure 23 compares the US firm size distribution to a power law. Although not perfect, the fit is good enough for modeling purposes. I model the US firm size distribution using a discrete power-law distribution with $\alpha = 2.01$.

A characteristic property of power-law distributions is that as α approaches 2, the average becomes *undefined*. This means that the hierarchy model can produce firm sizes that are extremely large — far beyond anything that exists in the real world. To stop this from happening, I *truncate* the power-law distribution at a maximum firm size of 2.3 million. This is the present size of Walmart, the largest US firm that has ever existed.

Code for the discrete power law random number generator can be found in `rpld.h`, located in the Supplementary Material. This code is an adaptation of Collin Gillespie's (2014) discrete power law generator found in the R poweRlaw package. Gillespie's generator is, in turn, an adaptation of the algorithm outlined by Clauset (2009).

E.2.2 Simulate the Employment Hierarchy

As in step 1, the employment hierarchy in modeled firms is assumed to be the same as the average in case-study firms (Fig. 17). But rather than receive CEO firm-size data as an input, the model now receives simulated data from a power law. All other steps for generating the employment hierarchy are then the same as in step 1.

E.2.3 Simulate the Distribution of Base-Level Pay

The next step is to simulate the hierarchical pay structure of each firm. To do this, we need to know the distribution of base-level pay. In step 1, we estimated

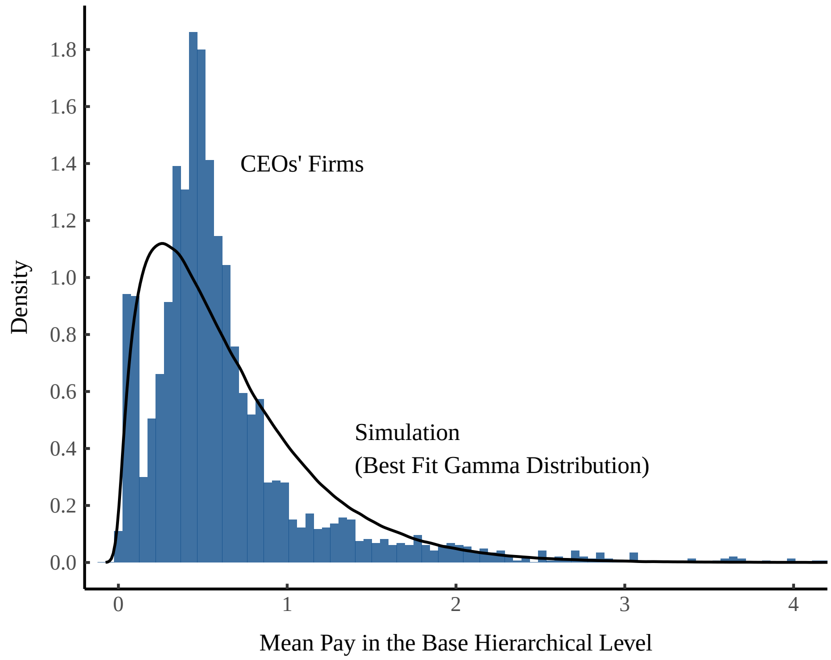

Figure 24: Modeling the Distribution of Base Pay in CEOs' Firms

This figure shows the distribution of (fitted) pay in the base level of CEO's firms. Pay is normalized so that the average income in the CEO sample (in each year) is 1. To extrapolate this data, I fit it with a gamma distribution. I do not attempt to replicate the bimodal structure of the CEO data, since it is likely not representative of the general firm population. The lower mode is composed mostly of chain restaurants, which seem to be over-represented in this sample.

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base-level pay in each CEO firm. In step 2, we fit a probability distribution to this data so that we can extrapolate it (Fig. 24).

I model the base-pay distribution in CEOs' firms with a gamma distribution. Because the CEO data has a bimodal structure, the gamma distribution is not a particularly strong fit, but it is better than other parameterized distributions. I do not attempt to replicate the bimodal structure of the CEO data because I feel it is not representative of the US firm population. The lower mode in CEO data is composed mostly of chain restaurants, which seem to be over-represented in this sample. While the shape of the fit is not great, the gamma distribution closely replicates the dispersion of base pay in the CEO sample. It reproduces the Gini index of roughly 0.35.

In each iteration, the model fits a gamma distribution to the CEO data. The model then samples from this distribution to simulate base pay in each firm generated by the power-law distribution. Code implementing this method is located in `base_pay_sim.h` in the Supplementary Material.

E.2.4 Simulate the Distribution of Hierarchical Pay

In step 1, we fitted the parameter r to each CEO firm. This parameter determines how rapidly income increases with hierarchical rank. In step 2, we extrapolate this information to a power-law sample of firms. To do this we fit the CEO distribution of r with a probability distribution. But unlike with base pay, the distribution of r depends on firm size.

As shown in Figure 25A, the dispersion in CEO r declines with firm size. I model this dispersion using the lognormal variate r_0:

$$r = 1 + \ln \mathcal{N}(r_0) \tag{40}$$

Here r_0 depends on firm size:

$$r_0(E) = \ln \mathcal{N}(r_0; \mu, \sigma_E) \tag{41}$$

The parameter of interest is σ_E. This determines the dispersion in r_0 for a given firm size E. To model r, we need to know how σ_E varies with firm size. To measure this, we first transform CEO r values using:

$$r_0 = r - 1 \tag{42}$$

This gives the CEO distribution of r_0. We estimate μ using:

$$\mu = \overline{\ln(r_0)} \tag{43}$$

We then estimate σ_E by taking the standard deviation of $\ln(r_0)$. But because σ_E varies by firm size (E), we calculate σ_E on groups of firms binned by firm size:

$$\sigma_E = \text{SD}[\ \ln(r_0)\ ]_E \tag{44}$$

Figure 25B plots σ_E vs. firm size for log-spaced bins of CEO firms. As expected, σ_E declines with firm size. I model this relation as:

$$\sigma_E \propto -\log(E) \tag{45}$$

Using this function, Figure 25C shows how the modeled dispersion in r_0 varies with firm size.

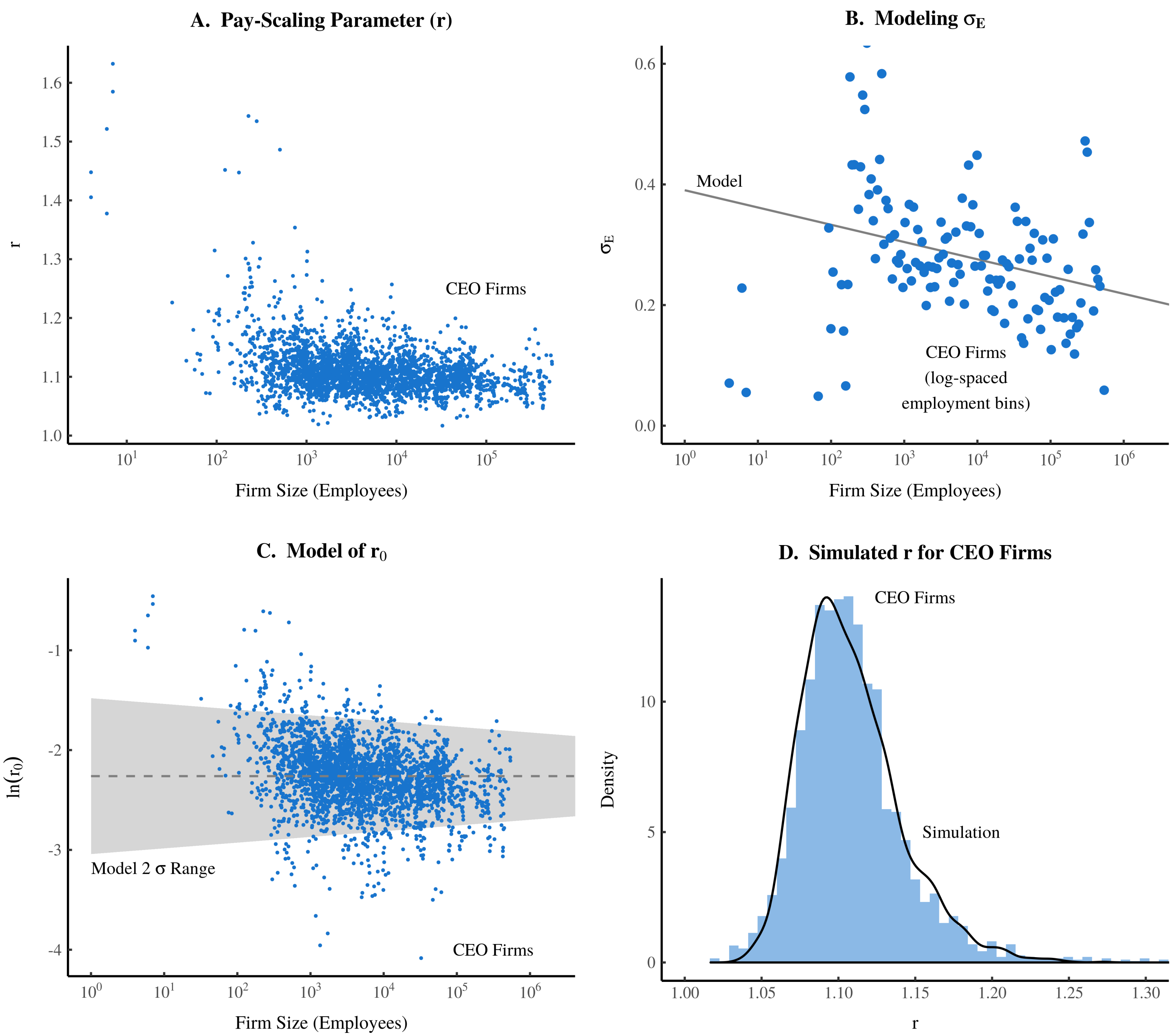

Figure 25: Modeling the Hierarchical Pay-Scaling Parameter

This figure visualizes the algorithm used to model the distribution of the parameter r. This parameter determines how rapidly income increases with hierarchical rank. Panel A shows the relation between r and firm employment within CEO firms. I simulate this relation using the lognormal variate r_0 (Eq. 41), which has a scale parameter σ_E that varies by firm size. Using Eq. 44, Panel B estimates σ_E for CEO firms. Each dot indicates σ_E for the given firm-size bin. The straight line indicates the modeled relation. Panel C shows how the modeled dispersion of $\ln(r_0)$ declines with firm size, and how this relates to CEO r_0 data. The 2σ range indicates 2 standard deviations from the mean (on log-transformed data). Panel D compares the distribution of r for CEO firms to the *simulated* distribution created by applying the model to the same CEO firms.

After estimating μ and σ_E from CEO data, the model uses Eq. 40 and 41 to determine r for each simulated firm (drawn from a power-law distribution). To test the accuracy of this algorithm, we can apply it back to CEO firms. For each CEO firm, we use the above algorithm to randomly generate values for r. As Figure 25D shows, the resulting distribution of r accurately reproduces the original data.

When we move from simulating CEO firms to a power-law distribution of firms, we undertake a significant extrapolation. This is because the CEO firm sample poorly represents small firms, so we have very little idea how r behaves among this small-firm population. As with all extrapolations, we do the best we can with the available data.

The code implementing this model is located in `r_sim.h` in the Supplementary Material.

E.2.5 Income Dispersion Within Hierarchical Ranks of Each Firm

So far, the model has simulated *average* pay by hierarchical rank. The last step is to add pay dispersion *within* ranks. To do this, we return to our firm case studies. Figure 16C shows the Gini index within each rank of each case-study firm. The model aims to replicate this data.

I model income dispersion within hierarchical ranks using a lognormal variate. The parameter σ determines the amount of inequality. The model assumes that σ is constant for all hierarchical ranks within all firms. The model estimates σ from case-study data. It first calculates $\bar{G}$ — the average Gini index of income dispersion within ranks. It then calculates the σ that would produce $\bar{G}$:

$$\sigma = 2 \cdot \mathrm{erf}^{-1}(\bar{G}) \tag{46}$$

This equation is derived from the definition of the Gini index of a lognormal distribution: $G = \mathrm{erf}(\sigma/2)$.

Because the case-study sample size is small, there is considerable uncertainty in σ. I quantify this uncertainty using the *bootstrap* method (Efron and Tibshirani, 1994). For each model iteration, I sample the case-study data (with replacement) and then estimate the parameter σ from this resampled data. Figure 26 shows the uncertainty in σ. Code implementing this method is located in `boot_sigma.h` in the Supplementary Material.

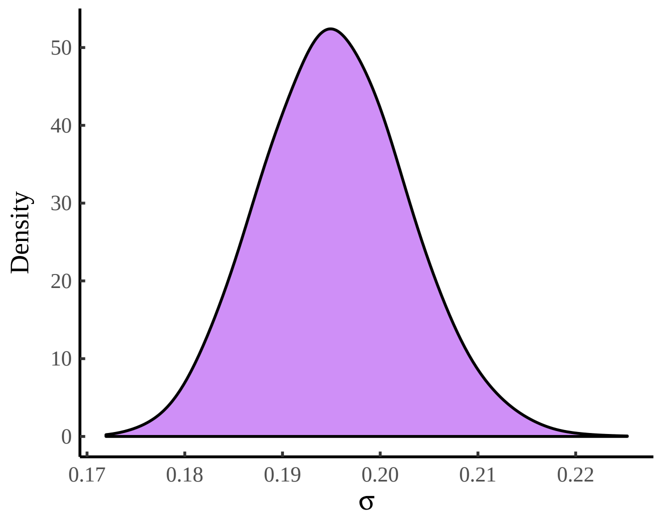

Figure 26: Uncertainty in the Parameter σ

This figure shows the uncertainty in the lognormal scale parameter σ, which determines pay dispersion within all hierarchical levels of all firms. The uncertainty is calculated using the bootstrap method.

E.3 Step 3: Simulate the Class Composition of Individual Income

The last step of the hierarchy model is to assign class components to individual income. The model does so by simulating the relation between income size and class found among US CEOs (Fig. 8A). The capitalist fraction of individual income (K_{frac}) is modeled as a function of hierarchical power (P):

$$K_{\text{frac}} = \kappa \log(P) \tag{47}$$

The capitalist gradient parameter κ determines the rate of this scaling and is fixed by regressions on CEO data. The model defines the labor fraction of individuals' income (L_{frac}) as the *complement* of the capitalist fraction:

$$L_{\text{frac}} = 1 - K_{\text{frac}} \tag{48}$$

Code implementing this method is located in `k_func.h` in the Supplementary Material.

Table 16: Structure of the Hierarchy Model

Step	Action	Reference Section	Parameter(s)	File(s)
1.1	Bootstrap firm case-study data	E.1.1, E.2.5	a, b, σ	`boot_span.h` `boot_sigma.h`
1.2	Estimate base-level employment in CEOs' firms	E.1.1	E_1	`base_fit.h`
1.3	Fit hierarchical pay-scaling parameter for each CEO firm	E.1.2	r	`fit_model.h`
1.4	Estimate base-level pay in each CEO firm	E.1.2	$\bar{I}_1$	`fit_model.h`
2.1	Generate a firm size distribution that follows a power law	E.2.1	α	`rpld.h`
2.2	Estimate base-level employment in each simulated firm	E.1.1	E_1	`base_fit.h`
2.3	Model the distribution of base-level pay. Assign a value to each simulated firm	E.2.3	$\bar{I}_1$	`base_pay_sim.h`
2.4	Model the distribution of the hierarchical pay-scaling parameter. Assign a value to each simulated firm	E.2.4	r	`r_sim.h`
2.5	Run hierarchy model	D	all but κ	`model.h`
3	Assign class composition to individual income	E.3	κ	`k_func.h`

Notes: Model code makes extensive use of Armadillo, an open-source linear algebra library for C++ (Sanderson and Curtin, 2016).

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