# Appendices
# for
# *Personal Income and Hierarchical Power*

Supplementary materials for this paper are available at the Open Science Framework repository:

https://osf.io/en4rz/

The supplementary materials include:

1. Data for all figures appearing in the article;
2. Raw source data;
3. R code for all analysis;
4. Compustat model code.

# Contents

# A Data Sources

## Age

Age mean income and within-group Gini index data is from US Census Tables PINC-02 over the years 1994-2015. Age is grouped into the following 4 categories: 18-24, 25-44, 45-64, 65 and older.

## Census Blocks

Census blocks data comes from the US Census American Community Survey (ACS) over the years 2010-2014. This data is tabulated at the *household* (rather than individual) level. Neither mean household income nor household Gini index data is directly available from the ACS at the census block level. I calculate mean household income by dividing aggregate household income by the number of households.

Within-group Gini indexes are estimated from binned income data using the R 'binequality' package. I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit either a lognormal or gamma distribution (whichever is best) to the binned data. Gini indexes are then calculated from this fitted distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R binequality package).

## Census Tracts

Census tract data comes from the US Census American Community Survey (ACS) over the years 2010-2015. Mean income data comes from series S1902, while within-group Gini indexes come from series B19083.

## Cognitive Score

The signal-to-noise ratio for cognitive score is estimated using data from Figure 6 in Bowles et al. (2001). Bowles' figure presents 65 different estimates (from 24 studies between 1963 and 1992) of the relation between individual income and cognitive score. The strength of this relation is quantified using the beta coefficient ($\beta$) of a log-linear regression. This coefficient represents the slope of the regression equation shown in Eq. 1, where the logarithm of income – $\log(I)$ – and cognitive score ($S$) have first been normalized to have a mean of 0 and standard deviation of one.

$$\log(I) = \alpha + \beta S \tag{1}$$

I use Engauge Digitizer to extract data from Bowles' graph. I then use a model to estimate the signal-to-noise ratio from Bowles' reported beta coefficients. The model creates a stochastic log-linear scaling relation between income and cognitive score. By adjusting the strength of this relation, we can create modeled data that has an equivalent beta coefficient to any of the points in Bowles' figure. I then use the model to calculate the signal-to-noise ratio for this beta coefficient.

The model assumes that cognitive score ($S$) is a normally distributed random variate with a mean of 100 and standard deviation of 15:

$$S \sim \mathcal{N}(100, 15) \tag{2}$$

We assume that the natural log of mean income ($\ln \bar{I}$) scales exponentially with cognitive score (Eq. 3). Since there is no evidence that extreme IQs lead to extreme incomes (at either the bottom or top end), I do not include them in the model. I model only those individuals with scores that are within two standard deviations of the mean ( $70 < S < 130$ ). The parameter $a$ determines how strongly cognitive score affects average income.

$$\ln(\bar{I}) = a(S - 70) \quad \text{for} \quad 70 < S < 130 \tag{3}$$

We assume that individual income ($I$) is a stochastic variable that is distributed according to a lognormal distribution defined by the location parameter $\mu$ and scale parameter $\sigma$:

$$I \sim \ln \mathcal{N}(\mu, \sigma) \tag{4}$$

Equation 5 shows how mean income $\bar{I}$ is related to $\mu$ and $\sigma$.

$$\bar{I} = e^{u + \frac{1}{2}\sigma^2} \tag{5}$$

By taking the logarithm and solving for $\mu$, Eq. 5 can be transformed into the following:

$$\mu = \ln(\bar{I}) - \frac{1}{2}\sigma^2 \tag{6}$$

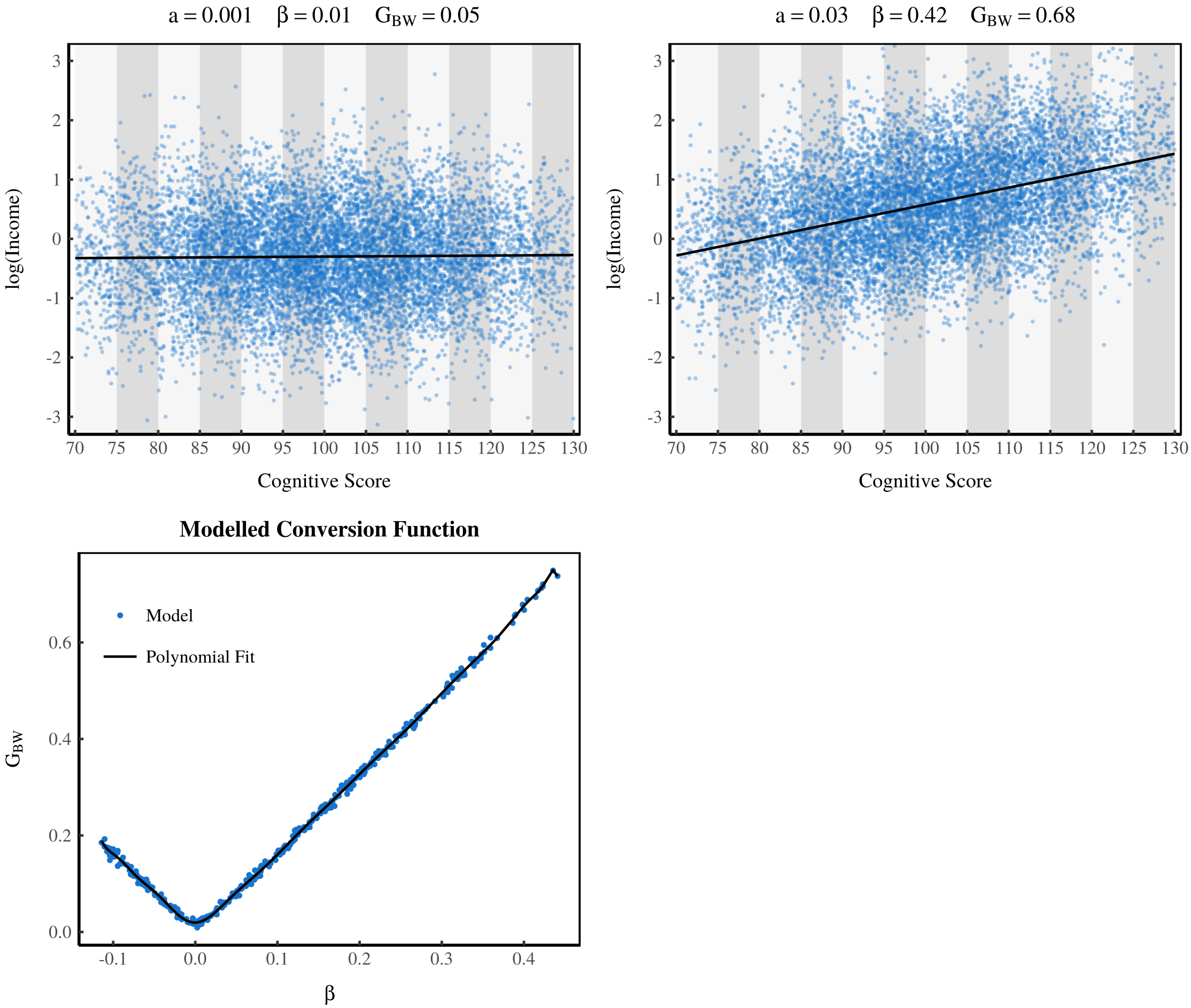


**Figure 1: Cognitive Score Method — Estimating the Signal-to-Noise Ratio from Normalized Regression Coefficients**

This figure shows an example of the model for converting cognitive score regression data from Bowles et al. (2001) to a signal-to-noise ratio. The signal-to-noise ratio ($G_{BW}$) is the ratio of the between-group Gini index to the average within-group Gini index. Using equations 2-7, I create a stochastic scaling relation between the logarithm of individual income and cognitive score. The strength of this scaling relation is determined by the parameter $a$, and is quantified by the normalized regression coefficient $\beta$. The top left panel shows a weak scaling relation, while the top right shows a strong scaling relation. I then group individuals into cognitive score intervals of 5 (vertical grey bars) and calculate signal-to-noise ratio ($G_{BW}$). The bottom left panel shows the resulting relation between $G_{BW}$ and $\beta$ that is used to convert Bowles' data.

We then substitute Eq. 3 into Eq. 6 to define $\mu$ in terms of cognitive score:

$$\mu = a(S - 70) - \frac{1}{2}\sigma^2 \tag{7}$$

The algorithm for the model is as follows. We first generate a random cognitive score $S$, drawn from the normal distribution defined by Eq. 2. We then take this score and use Eq. 7 to define the parameter $\mu$. Finally, we generate a random income for this cognitive score, drawn from the lognormal distribution defined by Eq. 4. This process is then repeated as many times to generate a stochastic dataset relating income to cognitive score.

The model has 2 free parameters: $a$ and $\sigma$. Parameter $a$ affects the rate at which income scales with cognitive score, while $\sigma$ determines the amount of dispersion around the mean income $\bar{I}$. The parameter $\sigma$ strongly affects the level of 'global' inequality in the model, while $a$ has only a slight effect. For this reason, it is important to chose $\sigma$ such that the model has a realistic level of inequality. I chose $\sigma = 0.8$. Over the chosen range of $-0.007 < a < 0.03$, this produces global Gini indexes that range between 0.43 and 0.47, which is roughly consistent with US data for the second half of the 20th century.

For any given value of $a$, the model generates a stochastic relation between cognitive score and income $I$. Two examples are shown in Figure 1. In Figure 1A, the small value of $a$ produces a very weak relation between income and cognitive score. In Figure 1B, the larger value of $a$ produces a stronger relation between income and cognitive score.

The strength of the relation is indicated by the beta coefficient $\beta$. The purpose of this model is to convert the values of $\beta$ reported by Bowles et al. into the signal-to-noise ratio that is used in this paper. To make this conversion, we must group individuals by their cognitive score. The bin-size of this grouping is arbitrary; I construct groupings of 5 point cognitive score intervals (indicated by the grey vertical bands in Fig. 1A-B). For each group, we calculate the mean income and within-group Gini index. The signal-to-noise ratio ($G_{BW}$) is then calculated by the method outlined in the main paper.

I repeat this process for many different values of $a$, which produces the modeled relation between $G_{BW}$ and $\beta$ shown in Figure 1C. I then fit this relation with a high order polynomial that serves as the function for converting Bowles' $\beta$ values into the signal-to-noise values used in this paper.

## Counties

US County data comes from the American Community survey for the years 2006-2015. County Gini indexes are from series B19083, while mean income is from series S1902.

## Education

Mean income and within-group Gini indexes by educational level come from US Census tables PINC-03 over the years 1994-2014. Educational level is categorized into the following groups:

- Less Than 9th Grade
- 9th to 12th Nongrad
- High school Graduate (Incl GED)
- Some College
- Associate Degree
- Bachelor's Degree
- Master's Degree
- Professional Degree
- Doctorate Degree

## Employees vs. Self-Employment

To calculate mean income and within-group Gini indexes for employees and self-employed workers, I use US Census table PINC-07 between 1994 and 2015. This table contains three categories: *Government Wage And Salary Workers*, *Private Wage And Salary Workers*, and *Self-Employed Workers*. Table 1 shows how I have mapped these categories onto the 'employees' and 'self-employed' sectors.

**Table 1: Grouping Categories of Census Table PINC-07**

| Employees | Self-Employed |
|---|---|
| • Government Wage And Salary Workers<br>• Private Wage And Salary Workers | • Self-Employed Workers |

Self-employed mean income and within-group Gini index come directly from PINC-07. To calculate the mean income of employees, I use the average of the means of government workers and private workers, weighted by the size of each group.

Since Gini indexes are not additive, I estimate the inequality among employees from binned data. I first add the binned income counts of both government and private wage/salary workers to get a binned income distribution for all 'employees'.

From this binned data, I then use the the R 'binequality' package to estimate private sector Gini indexes.

I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit various theoretical distributions to the binned data. Gini indexes are then calculated from the best-fitting distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R binequality package).

### Firms

Firm signal-to-noise ratio calculations use the Compustat database, and are a combination of empirical and modeled data. Firm mean income is calculated directly from Compustat data by dividing Total Staff Expenses (series XLR) by the number of employees (series EMP). Firm internal inequality is estimated using the Compustat Model. See Appendix B-G for a detailed discussion.

### Full and Part Time Workers

Full and part time worker mean income and within-group inequality data comes from US Census tables PINC-05 from 1994-2015.

### Parent Income Percentile

'Parent income percentile' refers to grouping individuals by the income percentile of their parents. My calculations are done using Table 1 and 2 from the online data tables of Chetty et al. (2014) — a seminal study of US intergenerational mobility. For every parent income percentile $x$, Table 1 gives the probability $p(x, y)$ that the corresponding child will have an income in percentile $y$. Table 2 gives the mean income ($\bar{I}_y$) of each child percentile $y$.

My method for estimating group mean incomes and within-group inequality is shown in equations 8 and 9. The first step is to convert the probability $p(x, y)$ into an integer $w(x, y)$ that can be used to weight incomes. Since the probabilities in Table 1 contain 7 decimal places, I multiply $p(x, y)$ by $10^7$ (Eq. 8).

$$w(x, y) = p(x, y) \times 10^7 \tag{8}$$

For each each income percentile $x$, we then create a vector of child incomes ($\mathbf{I_x}$)

by repeating each child percentile mean income $\bar{I}_y$ by the weighting factor $w(x, y)$. Here the notation $\overset{\times w(x,y)}{\cdots\cdots}$ indicates that the value $\bar{I}_y$ is repeated $w(x, y)$ times.

$$\mathbf{I_x} = \left( \bar{I}_1 \overset{\times w(x,1)}{\cdots\cdots}, \ \bar{I}_2 \overset{\times w(x,2)}{\cdots\cdots}, \ldots, \ \bar{I}_{100} \overset{\times w(x,100)}{\cdots\cdots} \right) \tag{9}$$

We can think of $\mathbf{I_x}$ as an estimated income distribution for children of parents in income percentile $x$. Mean income and within-group inequality of parent group $x$ are then estimated by calculating the mean and Gini index (respectively) of $\mathbf{I_x}$.

Note that this method neglects the income dispersion *within* each child income percentile (Chetty et al. do not provide this data). Thus, our estimated Gini index will have a slight *downward* bias.

## Hierarchical Level — Heyman

This data comes from Fredrik Heyman's (2005) study of 560 Swedish firms in the year 1995. His dataset includes only the top 4 levels of management. I include Heyman's results in the paper with the caveat that his data does not represent all hierarchical levels.

Heyman (Table A.1) provides the mean and standard deviation of the *logarithm* of incomes in each level. I estimate mean income ($\bar{I}$) and Gini index ($G$) by hierarchical level by assuming that intra-hierarchical level income is lognormally distributed. Under this assumption, the *mean* of log income is equal to the lognormal location parameter $\mu$, while the *standard deviation* of log income is equal to the scale parameter $\sigma$. Equations 10 and 11 then define the mean income and Gini index (respectively) of each hierarchical level.

$$\bar{I} = e^{\mu + \frac{1}{2}\sigma^2} \tag{10}$$

$$G = \text{erf}\left(\frac{\sigma}{2}\right) \tag{11}$$

Figure 2 shows how the implied aggregate inequality within the Heyman's sample compares to Swedish empirical data. Heymans's sample implies a bit less inequality than the empirical data. This is not surprising, however, as Heyman's data includes only the top 4 levels of management.

## Hierarchical Level — Mueller et al.

This data comes from Mueller et al. (2016), who study the hierarchical pay structure of 880 United Kingdom firms over the period 2004-2013. For each hierarchical

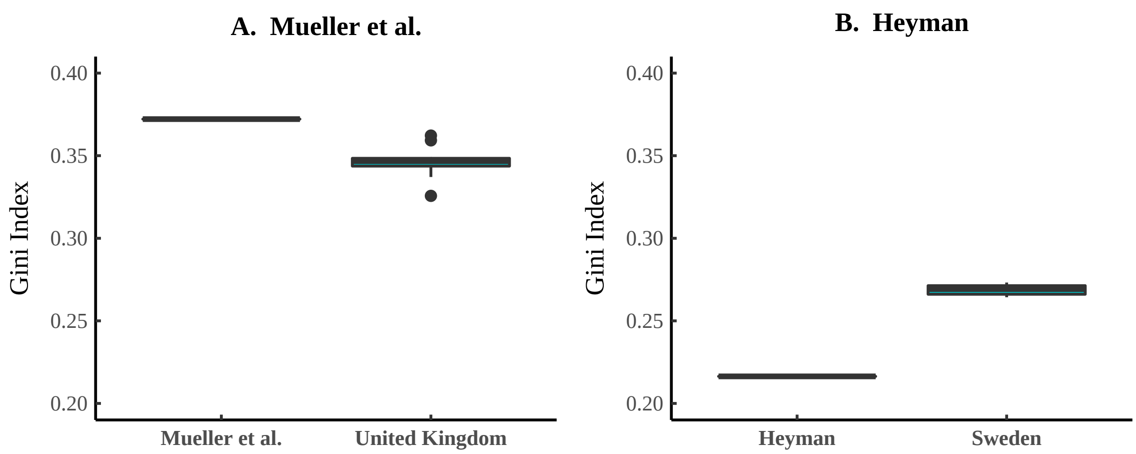


**Figure 2: Aggregate Inequality Implied by Hierarchy Data**

This figure compares levels of inequality implied by the Mueller et al. and Heyman firm samples against the inequality in their respective countries. UK inequality data is over the period 2004-2013, the same as covered by Mueller's data. Heyman's study covers the year 1995, while Swedish data is from 2004-2013. UK and Sweden Gini data is from the World Bank, series SI.POV.GINI.

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level, Mueller et al. provide the mean income as well as the 25th, 50th, and 75th income percentiles. To estimate intra-level inequality, I adapt R code written by Andrie de Vries to find the best-fit theoretical distribution for each hierarchical level. Intra-hierarchical level inequality is then calculated from the best-fit distribution.

Figure 2 shows how aggregate inequality within the Mueller et al. sample compares to UK data over the same period. Although the Mueller et al. data is slightly more unequal than the UK as a whole, it is a reasonably representative sample.

## Hierarchical Level — Compustat Model

The Compustat model is discussed extensively in Appendix B-G.

## Labor and Property Income

'Labor' income is defined as wages and salaries, while 'property' income is defined as the sum of interest, dividends, rents, royalties, and estates or trust income. Mean income and within-group inequality data comes from US Census tables PINC-08 from 2003-2015.

## Occupation

Data for mean income and with-group inequality by occupation comes from US Census tables PINC-06 (income by occupation of longest job) between 2007 and 2015. This table classifies occupations by major type, minor type, and detailed type. I use *detailed* categories only, which amounts to between 53 to 55 different occupation groups (depending on the year).

The US Bureau of Labor Statistics also publishes occupational wage estimates (available at https://www.bls.gov/oes/tables.htm). For the sake of completeness, I analyze this data here, but do not use it for the results published in the paper. The BLS data differs from Census data in the ways shown in Table 2.

Because the BLS does not report within-occupation Gini indexes directly, I estimate them via the reported values for 10th, 25th, 50th, 75th, and 90th income percentiles. Using an adaption of R code written by Andrie de Vries, I fit a variety of theoretical distributions to this percentile data. Within-occupation Gini indexes are calculated from the best-fit theoretical distribution.

The resulting signal-to-noise ratio is shown in Figure 3A, alongside the results from Census occupation data. The two calculations differ starkly. Census data indicates that between-occupation inequality is *less* than within-occupation inequality; however, the BLS data indicate the *reverse*.

Which result is correct? The answer to this question depends on the type of income inequality we are interested in explaining. The BLS data covers only full-time, non-self-employed workers earning labor income. Census data, on the other hand, includes *all* individuals. For the purposes of this paper, the Census data is a better choice.

**Table 2: Contrasting the US Census and BLS Occupational Income Data**

| **Census Data** | **BLS Data** |
|---|---|
| Includes self-employed workers | Does *not* include self-employed workers |
| Income for all full and part-time work | Income is for full-time equivalent workers only (hourly wage $\times$ 2080 hours). |
| Includes non-labor income | Does *not* include non-labor income |
| 53-55 detailed occupation types | 700-800 detailed occupational types |
| Reports Gini index directly | Reports 10th, 25th, 50th, 75th and 90th income percentiles |

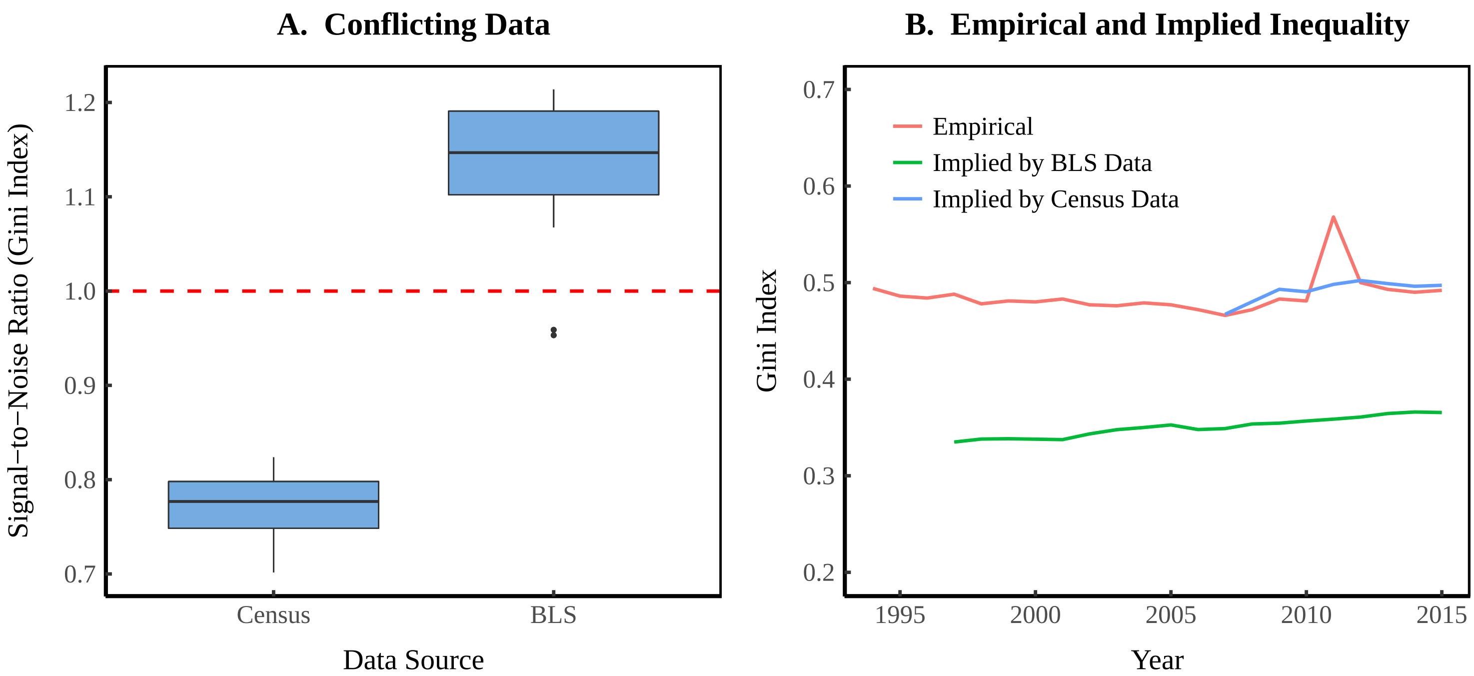


**Figure 3: Inequality by Occupation — Data Discrepancies**

This figure shows differences in the occupation income data published by the US Census versus that published by the US Bureau of Labor Statistics (BLS). Panel A shows calculations of the Gini index signal-to-noise ratio for both BLS and Census data. The BLS data gives a much higher signal-to-noise ratio, meaning between-occupation income dispersion is far greater (relative to within-occupation income dispersion) in BLS data than it is in the Census data. Why? The two datasets imply very different levels of aggregate (society-wide) inequality, as shown in panel B. This is because the BLS data includes only full-time wage/salary earners, while the Census data includes all individuals. The level of aggregate inequality implied by the Census data closely matches actual levels. I use Census data only in this paper.

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To demonstrate the differences between BLS and Census data, we can calculate the aggregate inequality that is implied by the data. To do this, I make the simplifying assumption that all occupations have lognormal income distributions. Given the mean income ($\bar{I}$) and within-group Gini index ($G$) of a particular occupation, we can define the lognormal location ($\mu$) and scale ($\sigma$) parameters:

$$\sigma = 2 \cdot \text{erf}^{-1}(G) \tag{12}$$

$$\mu = \ln(\bar{I}) - \frac{1}{2}\sigma^2 \tag{13}$$

If the number of individuals engaged in this occupation is $n$, we can create a simulated occupational income distribution by generating $n$ values from the lognormal distribution defined by $\mu$ and $\sigma$. We repeat this process for every occupation, and then aggregate all of the simulated occupational income distributions. The Gini index of this aggregated distribution is the level of inequality that is *implied* by the data.

The results of this analysis are shown in Figure 3B. As expected, the inequality that is implied by Census data closely matches actual levels of inequality between all individuals. However, the inequality implied by BLS data is *much* lower — a clear result of the restrictions underlying the BLS methods. For the purposes of this paper, the Census data is the correct choice.

## Owner vs. Renter

Mean income and within-group Gini indexes by home-ownership status come from US Census table PINC-01 between 1994 and 2015. I use the following two categories: (1) Owner Occupied; and (2) Renter Occupied.

## Public vs. Private Sector

To calculate mean income and within-group Gini indexes for public and private sector workers, I use US Census table PINC-07 between 1994 and 2015. This table contains three categories: *Government Wage And Salary Workers*, *Private Wage And Salary Workers*, and *Self-Employed Workers*. Table 3 shows how I have mapped these categories onto the 'public' and 'private' sectors.

**Table 3: Grouping Categories of Census Table PINC-07**

| Public Sector | Private Sector |
|---|---|
| • Government Wage And Salary Workers | • Private Wage And Salary Workers<br>• Self-Employed Workers |

The mean income and Gini index of the public sector is equivalent to the values for government wage/salary workers. Private sector mean income is calculated as the average of the means of private wage/salary worker income and self-employed worker income, weighted by the size of each group.

Since Gini indexes are not additive, I estimate the inequality of private sector income from binned data. I first add the binned income counts of both private wage/salary workers and self-employed workers to get a binned income distribution for the private sector. From this binned data, I then use the R 'binequality' package to estimate private sector Gini indexes.

I construct two different estimates: one using a parametric method and the other using the midpoint method. For the parametric method, I fit various theoretical distributions to the binned data. Gini indexes are then calculated from the best-fitting distribution. The midpoint method uses midpoints of the bins to estimate the Gini index. The midpoint of the upper bin (which has an open upper bound) is estimated from a best-fit power law (again, implemented in the R binequality package).

## Race

Data for mean income and within-group inequality by race comes from US Census tables PINC-01 between 1994 and 2015. Data for 2002-2015 contain the following four categories: Asian, Black, Hispanic, and White. Data for 1994-2001 contains only three categories: Black, Hispanic, and White.

## Religion

Religion income data comes from the Pew Research Center 2007 U.S. Religious Landscape Survey (RLS). I use the following groups:

- Agnostic
- Atheist
- Baptist
- Buddhist
- Church of Christ, or Disciples of Christ
- Congregational or United Church of Christ
- Episcopalian or Anglican
- Hindu
- Holiness (Nazarenes, Wesleyan Church, Salvation Army)
- Jewish
- Lutheran
- Methodist
- Mormon
- Muslim
- Nondenominational or Independent Church
- Nothing in particular
- Orthodox
- Pentecostal
- Presbyterian
- Reformed (include Reformed Church in America; Christian Reformed; Calvinist)
- Roman Catholic

The RLS reports the binned income of each respondent. I use the R 'binequality' package to estimate group mean income and Gini indexes (using the midpoint method). Because some religions have a very small sample size, I use the bootstrap method (Efron and Tibshirani 1994) to estimate a plausible range of values for group mean incomes and within-group income inequality.

### Sex

Data for mean income and within-group inequality by sex (male/female only) comes from US Census tables PINC-01 between 1994 and 2015.

### Urban vs. Rural

Data for urban/rural mean income and within-group Gini index comes from US Census tables PINC-01 between 1994 and 2015. I define 'urban' as individuals inside metropolitan statistical areas, and 'rural' as individuals outside these areas.

## B Hierarchical Structure and Pay Within Case-Study Firms

In this section I review the case-study evidence of firm hierarchy used in this paper. Table 4 summarizes the data sources, while Figure 4 shows the hierarchical employment and pay structure of these firms. The firms remain anonymous, and are named after the authors of the case studies. Although the exact shapes vary, all the firms in this sample have a roughly pyramidal employment structure and inverse pyramid pay structure.

Figure 5 dissects these trends to allow further analysis. Figure 5A shows how the span of control (the employment ratio between adjacent hierarchical levels) changes as a function of hierarchical level. In these firms, the span of control is not constant, but instead tends to *increase* with hierarchical level. Similarly, Figure 5B shows the ratio of mean pay between adjacent levels. Like the span of control, the pay ratio tends to increase with hierarchical level. Lastly, Figure 5C shows income dispersion within hierarchical ranks of each firm (measured with the Gini index). Note that income dispersion within levels is quite low and there is no evidence of a trend.

In addition to case-study data of single firms, several studies have reported the aggregate hierarchical structure of a sample of firms (see Table 5 and Figure 6). The data from these firms reveals the same general trends as the case studies. However, the aggregate data is less useful because these studies capture only the top few hierarchical ranks within firms.

From the case-study evidence, I propose the following 'stylized' facts about firm employment and pay structure:

1. The span of control tends to *increase* with hierarchical level.
2. The inter-level pay ratio tends to *increase* with hierarchical level.
3. Intra-level income inequality is approximately *constant* across all hierarchical levels.

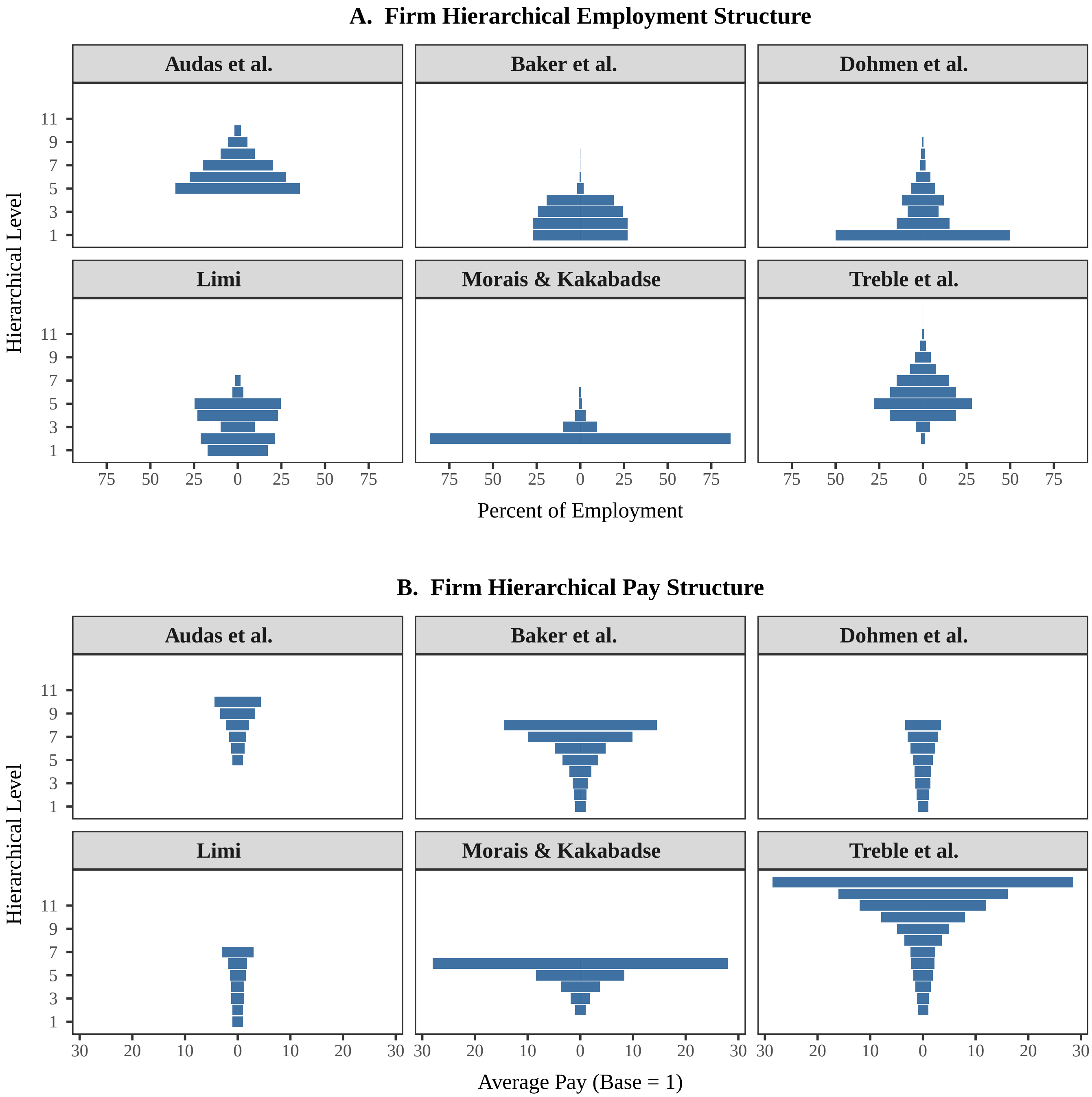


**Figure 4: The Hierarchical Employment and Pay Structure of Six Different Firms**

This figure shows the hierarchical employment and pay structure of six different case-study firms. Panel A shows the hierarchical structure of employment, while Panel B shows the hierarchical pay structure.

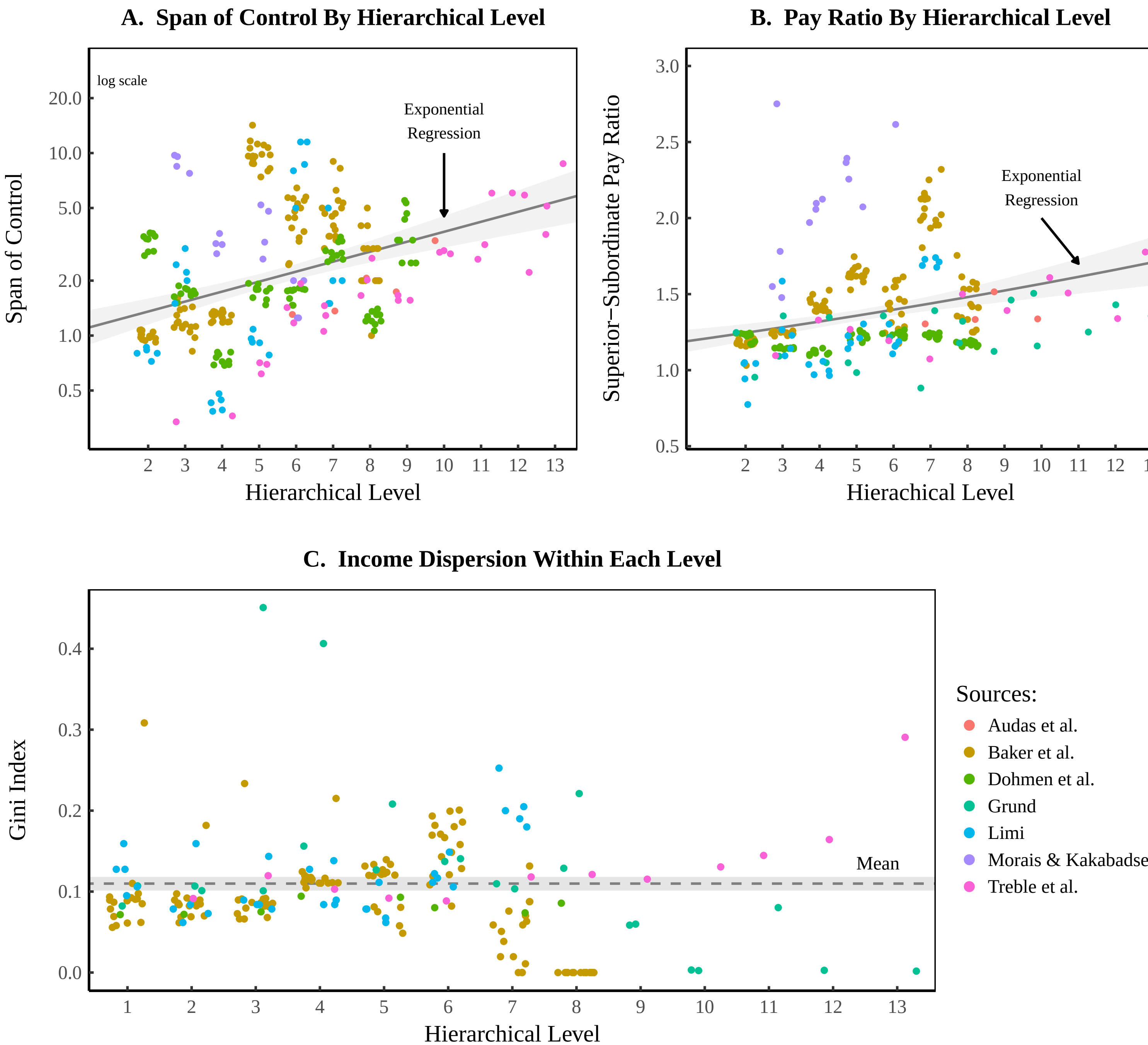


**Figure 5: Case Studies of Firm Hierarchical Structure**

This figure shows data from 7 case-study firms. Panel A shows how the span of control (the subordinate-to-superior employment ratio between adjacent levels) varies with hierarchical level. Note the log scale on the y-axis. Panel B shows how the superior-to-subordinate pay ratio varies with hierarchical level. In Panels A and B, the $x$-axis corresponds to the *upper* hierarchical level in each corresponding ratio. Panel C shows the Gini index of income inequality within each hierarchical level. Different case-study firms are indicated by color, with names indicating the study author. Note that horizontal 'jitter' has been introduced in all three plots in order to better visualize the data (hierarchical level is a discrete variable). The lines in Panels A and B indicate exponential regressions, while the line in Panel C shows the average Gini index. Grey regions correspond to the 95% confidence intervals.

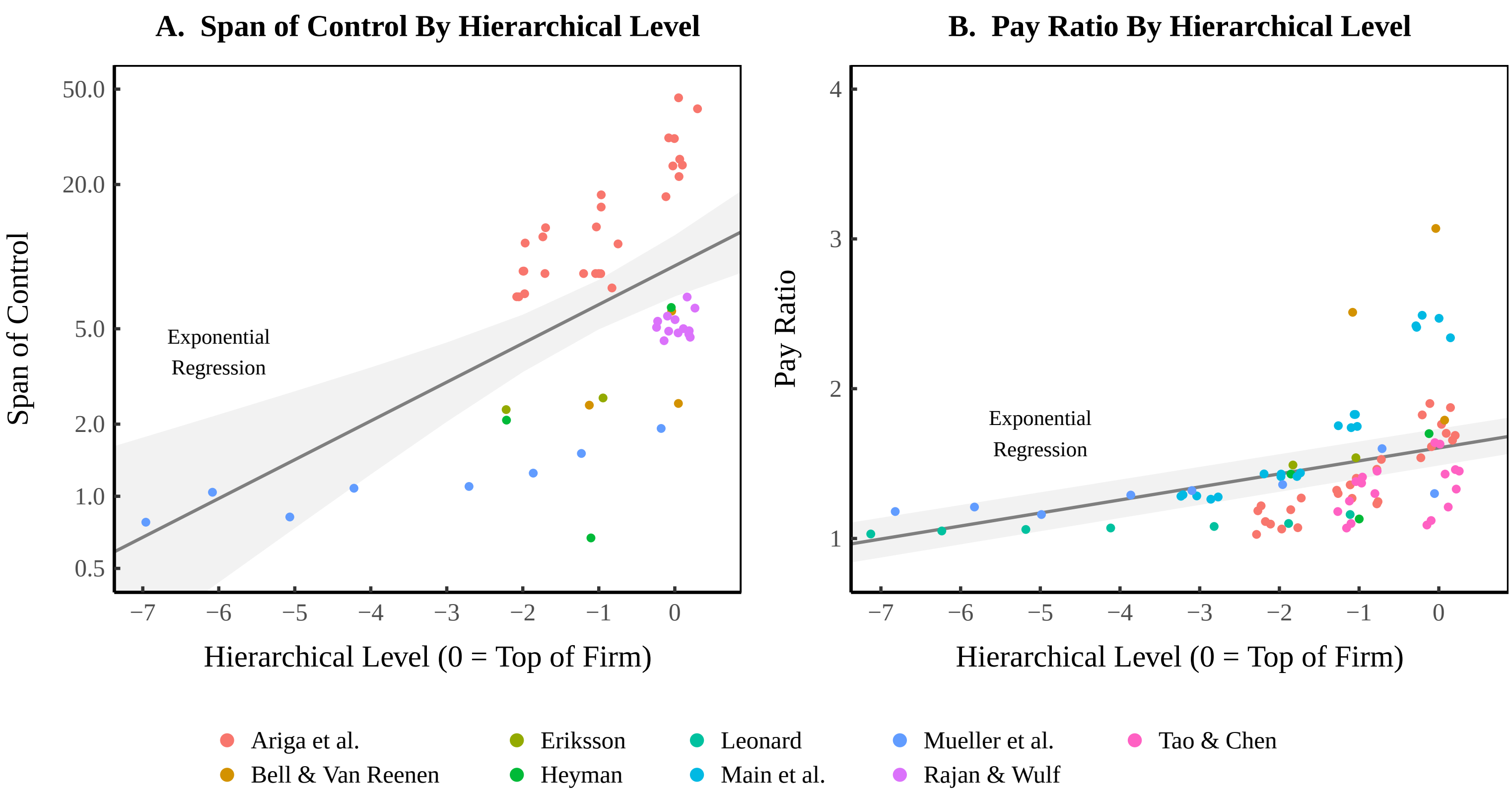


**Figure 6: Aggregate Studies of Firm Hierarchical Structure**

This figure shows data from 9 different aggregate firm studies. Most of these studies only survey the top several hierarchical levels in each firm. Because of this, I order hierarchical levels from the top down, where the CEO is level 0, the level below is -1, etc. Panel A shows how the span of control (the employment ratio between adjacent levels) relates to hierarchical level. Panel B shows how the pay ratio between adjacent levels varies with hierarchical level. In both plots, horizontal 'jitter' has been introduced in order to better visualize the data (hierarchical level is a discrete variable). Grey regions correspond to the 95% confidence interval for regressions.

**Table 4: Firm Case Studies**

| Source | Years | Country | Firm Levels | Span of Control | Level Income | Level Income Dispersion |
|---|---|---|---|---|---|---|
| Audas, Barmby, and Treble (2004) | 1992 | Britain | Grade 6–10 | ✓ | ✓ | |
| Baker, Gibbs, and Holmstrom (1993) | 1969-1985 | United States | Management | ✓ | ✓ | ✓ |
| Dohmen, Kriechel, and Pfann (2004) | 1987-1996 | Netherlands | All | ✓ | ✓ | ✓ |
| Grund (2005) | 1995 & 1998 | US and Germany | All | | ✓ | ✓ |
| Lima (2000) | 1991-1995 | Portugal | All | ✓ | ✓ | ✓ |
| Morais and Kakabadse (2014)* | 2007-2010 | Undisclosed | All | ✓ | ✓ | |
| Treble, Van Gameren, Bridges et al. (2001) | 1989-1994 | Britain | All | ✓ | ✓ | ✓ |

**Notes**: This table shows metadata for the firm case studies displayed in Fig. 5. 'Firm Levels' refers to the portion of the firm that is included in the study. 'Management' indicates that only management levels were studied.

*For the analysis conducted in this paper I discard (as an outlier) the bottom hierarchical level in Morais and Kakabadse's data.

**Table 5: Firm Aggregate Studies**

| Source | Years | Number of Firms | Country | Firm Levels | Span of Control | Level Income |
|---|---|---|---|---|---|---|
| Ariga, Brunello, Ohkusa et al. (1992) | 1981-1989 | unknown | Japan | All | ✓ | ✓ |
| Bell and Van Reenen (2012) | 2001-2010 | 552 | United Kingdom | Top 3 | ✓ | ✓ |
| Eriksson (1999) | 1992-1995 | 210 | Denmark | Management | ✓ | ✓ |
| Heyman (2005) | 1991,1995 | 560 | Sweden | Management | ✓ | ✓ |
| Leonard (1990) | 1981-1985 | 439 | United States | Top 9 | | ✓ |
| Main, O'Reilly III, and Wade (1993) | 1980–1984 | 200 | United States | Top 4 | | ✓ |
| Mueller, Ouimet, and Simintzi (2016) | 2004-2013 | 880 | United Kingdom | All | ✓ | ✓ |
| Rajan and Wulf (2006) | 1986-1998 | 261 | United States | Top 2 | ✓ | |
| Tao and Chen (2009) | 1986-1998 | 8101 | Taiwan | Top 2 | | ✓ |

**Notes**: This table shows metadata for the aggregate studies displayed in Fig. 6. 'Firm Level's refers to the portion of the firm that is included in the study. 'Top 2', 'Top 3', etc. indicates that only the top $n$ levels were included in the study (where the top level is the CEO).

**Table 6: Income Inequality Within Case Study Firms**

| Source | Years | Mean Gini Index |
|---|---|---|
| Baker, Gibbs, and Holmstrom (1993) | 1969-1985 | 0.32 |
| Dohmen, Kriechel, and Pfann (2004) | 1991 | 0.18 |
| Lima (2000) | 1991-1995 | 0.15 |
| Morais and Kakabadse (2014) | 2007-2010 | 0.23 |
| Treble, Van Gameren, Bridges et al. (2001) | 1989-1997 | 0.26 |

## B.1 Inequality Within Case Study Firms

I report here my estimates for inequality within the case study firms. Of the seven case studies summarized in Table 4, only one (Morais and Kakabadse) directly reports a firm Gini index. However, four other studies — Baker et al., Dohmen et al., Lima, and Treble et al. — provide enough data to allow estimates of firm internal inequality. I outline my calculation methods below. The resulting Gini estimates are shown in Table 6.

### Baker et al.

Baker et al. (1993) have made their raw personnel data publicly available at the site below. I use this raw data to calculate the firm internal Gini index.

http://faculty.chicagobooth.edu/michael.gibbs/research/index.html

### Dohmen et al.

Dohmen et al. (2004) report the following data that I use to estimate the firm Gini index:

1. Fraction of employment by hierarchical level (Tbl. 1);
2. Density plots of income distribution by hierarchical level (Fig. 5).

I use the *Engauge Digitizer* program to digitize and pull data from the density plots. I then use the resulting numerical density functions to estimate the firm Gini index.

We define $f_h(x)$ as the income density function for hierarchical level $h$. The income density function of the entire firm $f_T(x)$ is then defined by Eq. 14 – the sum of the density functions for each hierarchical level, weighted by the fraction of total employment ($E_h/E_T$).

$$f_T(x) = \sum_{h=1}^{n} \frac{E_h}{E_T} \cdot f_h(x) \tag{14}$$

The firm Gini index is then defined by Eq. 15-17. Equation 15 defines the mean income of the firm ($\bar{I}$), while equation 16 defines the cumulative income distribution function $F(x)$. Equation 17 then defines the Gini index ($G$). I use numerical integration implemented in R to evaluate these integrals.

$$\bar{I} = \int_0^\infty x \cdot f_T(x)\, dx \tag{15}$$

$$F(x) = \int_0^\infty f_T\, dx \tag{16}$$

$$G = \frac{1}{\bar{I}} \int_0^\infty F(x)(1 - F(x)) dx \tag{17}$$

### Grund

Grund (2005) does not provide enough information to calculate firm-wide inequality. However, I am able to calculate intra-level income dispersion, (which appears in Fig. 5C). I use data from Grund's Fig. 1, which shows mean income by level, as well as what I assume to be 5th and 95th percentiles. After digitizing this data, I use the best-fit theoretical distribution to estimate the Gini index.

### Lima

Lima (2000) provides the following summary statistics, which I use to estimate a firm Gini index:

1. Employment within each hierarchical level (Tbl. 1);
2. Mean pay within each hierarchical level (Fig. 2);
3. Wage coefficient of variation by hierarchical level (Tbl. 6).

I use the *Engauge Digitizer* program to digitize and pull data from Fig. 2. To calculate the firm Gini index, I assume income within each hierarchical level is lognormally distributed. For each hierarchical level $h$, I then use equation 18 to define the lognormal scale parameter $\sigma$ that produces a distribution with an equivalent coefficient of variation, $c_v$:

$$\sigma_h = \sqrt{\ln(c_v^2 + 1)} \tag{18}$$

Once we have $\sigma_h$, we use equation 19 to calculate the lognormal location parameter $\mu$ for each hierarchical level . Here $\bar{I}_h$ is the mean pay in hierarchical level $h$ (which Lima reports directly).

$$\mu_h = \ln(\bar{I}_h) - \frac{1}{2}\sigma_h^2 \tag{19}$$

Once we have the appropriate lognormal parameters for each hierarchical level, we use these distributions to create a simulated payroll. To do this, we draw $E_h$ numbers (employment in level $h$) from each lognormal distribution $\ln \mathcal{N}(\mu_h, \sigma_h)$. I then calculate the Gini index from this simulated payroll.

**Treble et al.**

Treble et al. (2001) report the following summary statistics, which I use to estimate a firm Gini index:

1. Employment within each hierarchical level (Fig. 2);
2. Mean pay within each hierarchical level (Fig. 3);
3. 5th and 95th wage percentile by hierarchical level (Fig. 4).

Again, I use *Engauge Digitizer* to pull data from all graphs. To estimate the intra-level Gini index, I adapt code writtent by Andrie de Vries to fit a parameterized distribution to the mean and 5th/95th percentiles.

# C The Compustat Data

To model the hierarchical structure of US firms, I use the Compustat data series shown in Table 7. Selected statistics from this dataset are shown in Figure 7. For each Compustat firm (in each year), I calculate the following statistics:

$$\text{Employee Mean Income} = \frac{\text{Total Staff Expenses}}{\text{Employees}} \tag{20}$$

$$\text{CEO Pay Ratio} = \frac{\text{Top Exec Pay}}{\text{Employee Mean Income}} \tag{21}$$

Note that 'CEO pay' is defined as the income of the top-paid executive in a given firm.

**Table 7: Compustat Data Series**

| Database | Series ID | Description |
|---|---|---|
| ExecuComp | TDC1 | Executive Total Compensation |
| Fundamentals Annual | XLR | Total Staff Expenses |
| Fundamentals Annual | EMP | Employees |

Notes: Executive compensation series TDC1 = Salary + Bonus + Other Annual + Restricted StockGrants + LTIPPayouts + All Other + Value of Option Grants

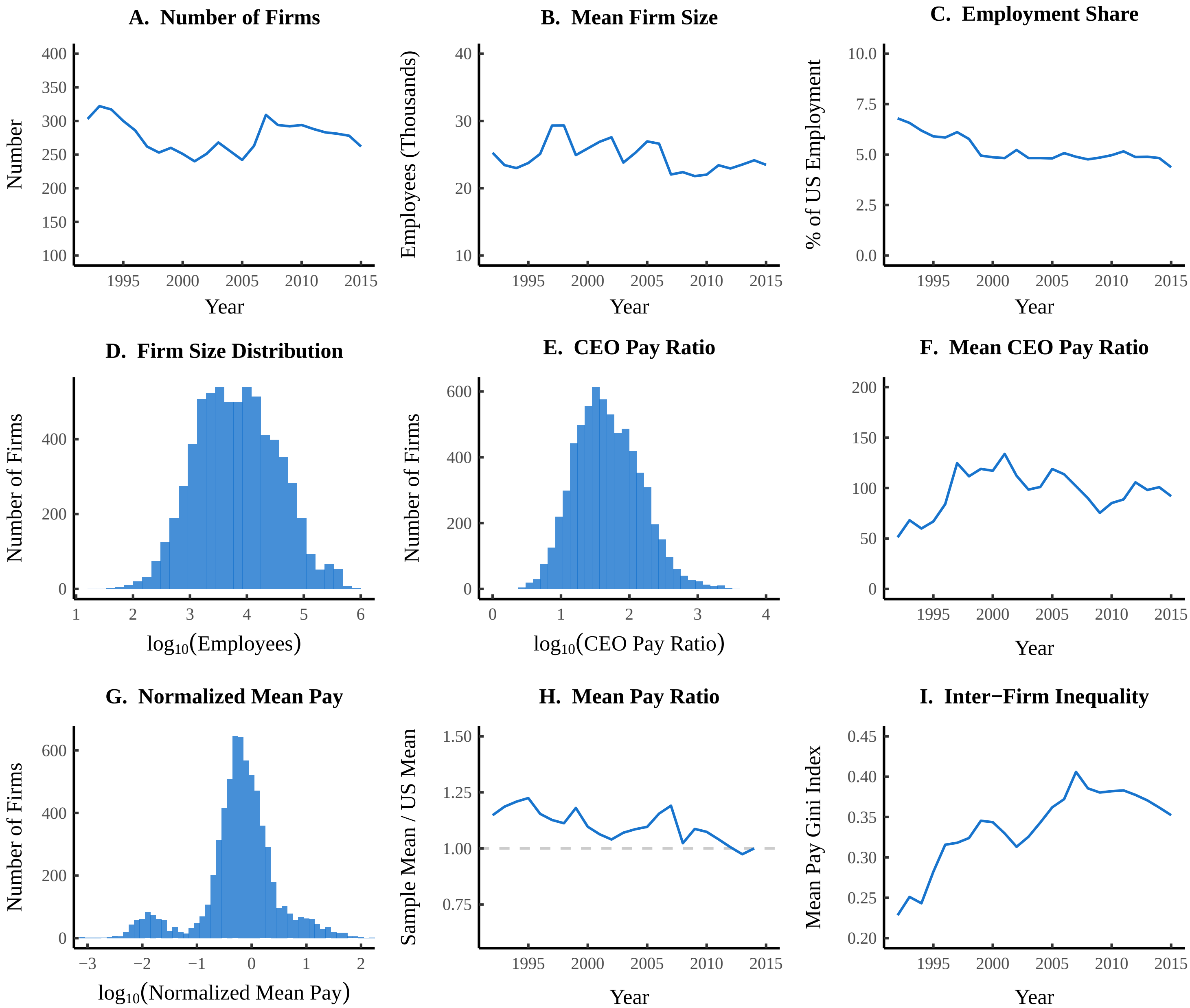


**Figure 7: Selected Statistics from the Compustat Firm Sample**

This figure shows statistics for the Compustat firm sample, which consists of US firms for which the data series in Table 7 are available. In panel H, US mean income per worker is calculated from national accounts (BEA Table 1.12, National Income by Type of Income) by dividing the sum of employee and proprietor income by the number of workers (BEA Table 6.8C-D, persons engaged in production).

## D A Hierarchy Algorithm

In this section, I outline the mathematics underlying my hierarchical model of the firm. The model assumptions, outlined below, are based on the stylized facts gleaned from the real-world firm data in section B.

1. Firms are hierarchically structured, with a span of control that increases *exponentially* with hierarchical level.
2. The ratio of mean pay between adjacent hierarchical levels increases *exponentially* with hierarchical level.
3. Intra-hierarchical-level income is lognormally distributed and constant across all levels.

Using these assumptions, I first develop an algorithm that describes the hierarchical employment within a model firm, followed by an algorithm that describes the hierarchical pay structure.

**Table 8: Notation**

| **Symbol** | **Definition** |
|---|---|
| $a$ | span of control parameter 1 |
| $b$ | span of control parameter 2 |
| $C$ | CEO to average employee pay ratio |
| $E$ | employment |
| $F$ | cumulative distribution function |
| $G$ | Gini index of inequality |
| $h$ | hierarchical level |
| $\bar{I}$ | average income |
| $\mu$ | lognormal location parameter |
| $n$ | number of hierarchical levels in a firm |
| $p$ | pay ratio between adjacent hierarchical levels |
| $r$ | pay-scaling parameter |
| $s$ | span of control |
| $\sigma$ | lognormal scale parameter |
| $T$ | total for firm |
| $\downarrow$ | round down to nearest integer |
| $\prod$ | product of a sequence of numbers |
| $\sum$ | sum of a sequence of numbers |

### D.1 Generating the Employment Hierarchy

To generate the hierarchical structure of a firm, we begin by defining the span of control ($s$) as the ratio of employment ($E$) between two consecutive hierarchical levels ($h$), where $h = 1$ is the *bottom* hierarchical level. It simplifies later calculations if we define the span of control in level 1 as $s = 1$. This leads to the following piecewise function:

$$s_h \equiv \begin{cases} 1 & \text{if } \; h = 1 \\ \dfrac{E_{h-1}}{E_h} & \text{if } \; h \geq 2 \end{cases} \tag{22}$$

Based on our empirical findings in Section B, we assume that the span of control is *not* constant; rather it increases *exponentially* with hierarchical level. I model the span of control as a function of hierarchical level ($s_h$) with a simple exponential function, where $a$ and $b$ are free parameters:

$$s_h = \begin{cases} 1 & \text{if } \; h = 1 \\ a \cdot e^{bh} & \text{if } \; h \geq 2 \end{cases} \tag{23}$$

As one moves up the hierarchy, employment in each consecutive level ($E_h$) *decreases* by $1/s_h$. This yields Eq. 24, a recursive method for calculating $E_h$. Since we want employment to be *whole* numbers, we round down to the nearest integer (notated by $\downarrow$). By repeatedly substituting Eq. 24 into itself, we can obtain a non-recursive formula (Eq. 25). In product notation, Eq. 25 can be written as Eq. 26.

$$E_h =\downarrow \frac{E_{h-1}}{s_h} \qquad \text{for} \qquad h > 1 \tag{24}$$

$$E_h =\downarrow E_1 \cdot \frac{1}{s_2} \cdot \frac{1}{s_3} \cdot \ldots \cdot \frac{1}{s_h} \tag{25}$$

$$E_h =\downarrow E_1 \prod_{i=1}^{h} \frac{1}{s_i} \tag{26}$$

Total employment in the whole firm ($E_T$) is the sum of employment in all hierarchical levels. Defining $n$ as the total number of hierarchical levels, we get Eq. 27, which in summation notation, becomes Eq. 28.

$$E_T = E_1 + E_2 + \ldots + E_n \tag{27}$$

$$E_T = \sum_{h=1}^{n} E_h \tag{28}$$

In practice, $n$ is not known beforehand, so we define it using Eq. 26. We progressively increase $h$ until we reach a level of zero employment. The highest level $n$ will be the hierarchical level directly *below* the first hierarchical level with zero employment:

$$n = \{h \quad | \quad E_h \geq 1 \quad \text{and} \quad E_{h+1} = 0\} \tag{29}$$

To summarize, the hierarchical employment structure of our model firm is determined by 3 free parameters: the span of control parameters $a$ and $b$, and base-level employment $E_1$.

## D.2 Generating Hierarchical Pay

To model the hierarchical pay structure of a firm, we begin by defining the inter-hierarchical pay-ratio ($p_h$) as the ratio of mean income ($\bar{I}$) between adjacent hierarchical levels. Again, it is helpful to use a piecewise function so that we can define a pay-ratio for hierarchical level 1:

$$p_h \equiv \begin{cases} 1 & \text{if} \ \ h = 1 \\ \dfrac{\bar{I}_h}{\bar{I}_{h-1}} & \text{if} \ \ h \geq 2 \end{cases} \tag{30}$$

Based on our empirical findings in Section B, we assume that the pay ratio increases *exponentially* with hierarchical level. I model this relation with the following function, where $r$ is a free parameter:

$$p_h = \begin{cases} 1 & \text{if} \ \ h = 1 \\ r^h & \text{if} \ \ h \geq 2 \end{cases} \tag{31}$$

Using the same logic as with employment (shown above), the mean income $I_h$ in any hierarchical level is defined recursively by Eq. 32 and non-recursively by Eq. 33.

$$\bar{I}_h = \frac{\bar{I}_{h-1}}{p_h} \tag{32}$$

$$\bar{I}_h = \bar{I}_1 \prod_{i=1}^{h} p_i \tag{33}$$

To summarize, the hierarchical pay structure of our model firm is determined by 2 free parameters: the pay-scaling parameter $r$, and mean pay in the base level ($\bar{I}_1$).

### D.2.1 Useful Statistics

Two statistics are used repeatedly within the model: mean firm pay, and the CEO-to-average-employee pay ratio. Mean income for all employees ($\bar{I}_T$) is equal to the average of hierarchical level mean incomes ($\bar{I}_h$) weighted by the respective hierarchical level employment ($E_h$):

$$\bar{I}_T = \sum_{h=1}^{n} \bar{I}_h \cdot \frac{E_h}{E_T} \tag{34}$$

To calculate the CEO pay ratio, we define the CEO as the person in the top hierarchical level. Therefore, CEO pay is simply $\bar{I}_n$, average income in the top hierarchical level. The CEO pay ratio ($C$) is then equal to CEO pay divided by average pay:

$$C = \frac{\bar{I}_n}{\bar{I}_T} \tag{35}$$

## D.3 Adding Intra-Level Pay Dispersion

Up to this point, we have modeled only the *mean* income within each hierarchical level of a firm. The last step in the modeling process is to add pay *dispersion* within each hierarchical level.

I assume that pay dispersion within hierarchical levels is *lognormally* distributed. The lognormal distribution is defined by location parameter $\mu$ and scale parameter $\sigma$. Our empirical investigation of firm case studies indicated that pay dispersion with hierarchical levels is relatively constant (see Fig. 5C). Given this finding, I assume *identical inequality* within all hierarchical levels. This means that the lognormal scale parameter $\sigma$ is the same for all hierarchical levels.

In order to add dispersion within each hierarchical level, I multiply mean pay $\bar{I}_h$ by a lognormal random variate with an expected mean of one. Formally, this is represented by Eq. 36. Since the mean of a lognormal distribution is equal to

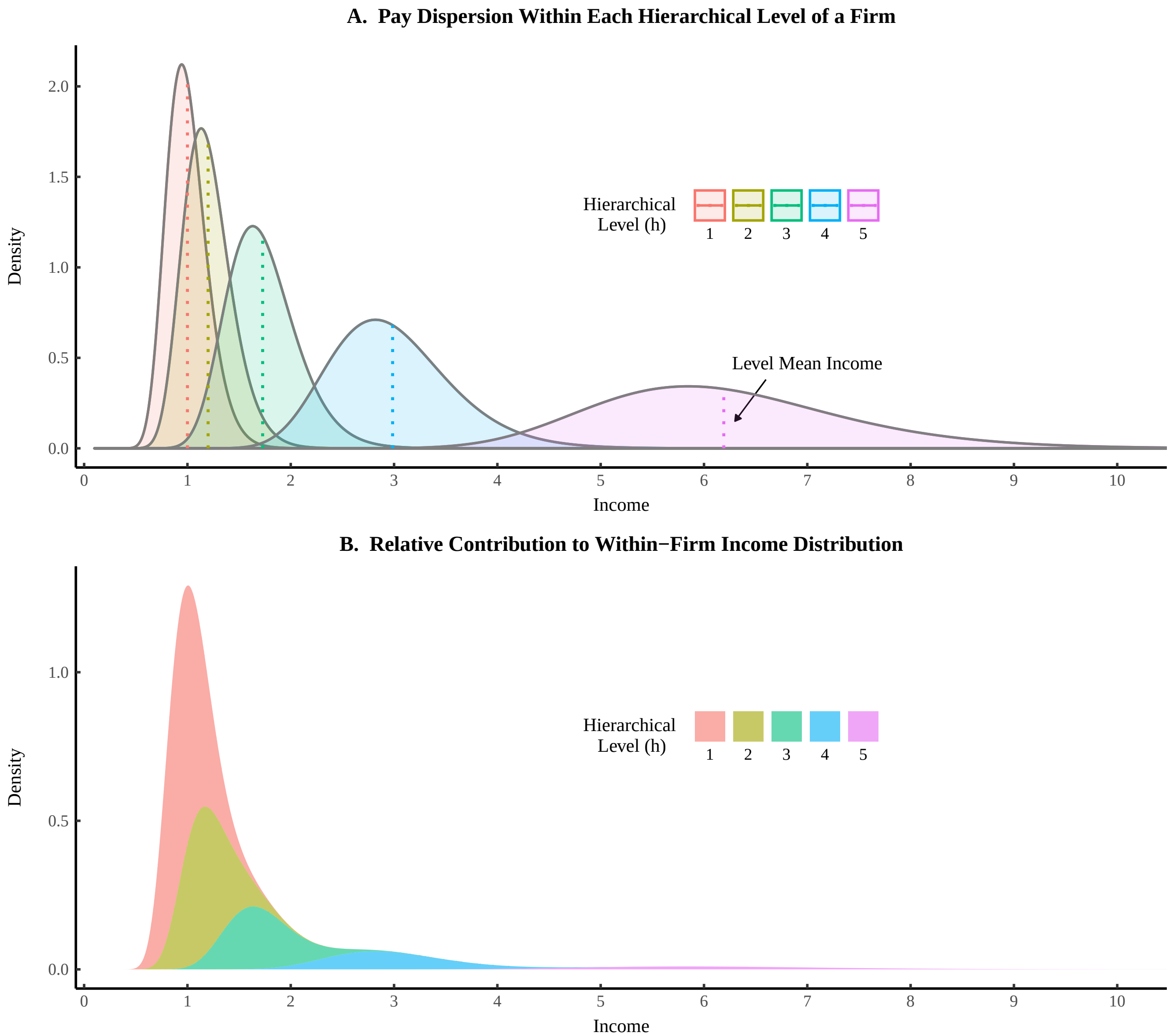


**Figure 8: Adding Intra-Level Pay Dispersion to a Model Firm**

This illustrates a model firm with lognormal pay dispersion in each hierarchical level. Panel A shows the separate distributions for each level, with mean income indicated by a dashed vertical line. Panel B shows contribution of each hierarchical level to the resulting income distribution for the whole firm (income density functions are summed while weighting for their respective employment).

$e^{\mu+\frac{1}{2}\sigma^2}$, I leave it to the reader to show that a mean of one requires that $\mu$ be defined by Eq. 37.

$$I_h = \bar{I}_h \cdot \ln \mathcal{N}(\mu, \sigma) \tag{36}$$

$$\mu = -\frac{1}{2}\sigma^2 \tag{37}$$

Given a value for $\sigma$ (which is a free parameter), we can define the pay distribution within any hierarchical level of a firm. This process is shown graphically in Figure 8. Figure 8A shows the lognormal income distributions for each hierarchical level of a 5-level firm. Figure 8B shows the size-adjusted contribution of each hierarchical level to the overall intra-firm income distribution. Lower levels have more members, and thus dominate the overall distribution.

## E The Compustat Model

The Compustat model takes the available firm case-study data and uses it to estimate the hierarchical structure of US firms in the Compustat database. The model assumes that each Compustat firm has an employment hierarchy shaped like Figure 9. This is the shape that is implied by fitting a trend to the span of control data for case-study firms (Fig. 5A). This restricting assumption allows us to use the available data on firm employment, mean pay, and CEO pay to estimate the hierarchical pay structure of each Compustat firm. The idea is that when the hierarchical employment structure of a firm is pre-specified (using case-study data), the CEO pay ratio allows us to calculate how income increases by hierarchical level. Of course, for any specific firm, this estimation method is likely not that accurate. However, the hope is that on average, this model provides insights into the hierarchical structure of Compustat firms.

The Compustat model uses the algorithms discussed in Appendix D with parameters summarized in Table 9. In the following sections, I outline how I restrict each model parameter.

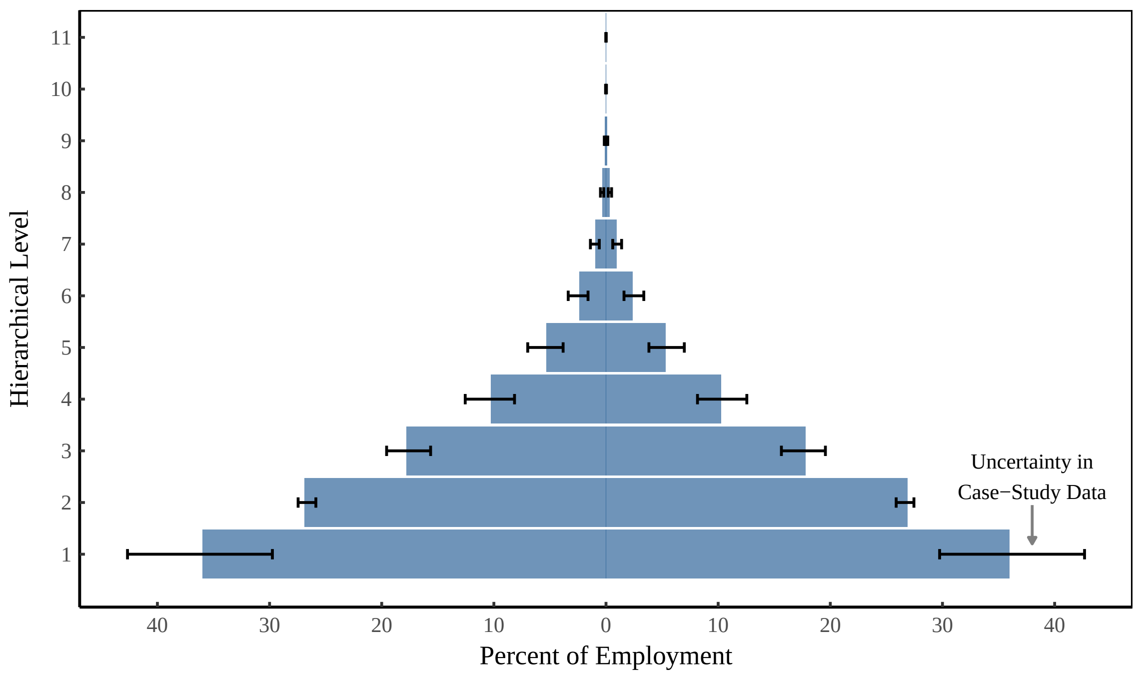


**Figure 9: Idealized Firm Employment Hierarchy Implied by Case Studies**

This figure shows the idealized firm hierarchy that is implied by fitting trends to case-study data (Fig. 5A). Error bars show the uncertainty in the hierarchical shape, calculated using a bootstrap resample of case-study data.

**Table 9: Model Parameters**

| Parameter | Definition | Action | Scope |
|---|---|---|---|
| $a$, $b$ | Span of control parameters | Determines the shape of the firm hierarchy. | Identical for all firms. |
| $E_1$ | Employment in base hierarchical level | Used to build the employment hierarchy from the bottom up. Determines total employment. | Specific to each firm. |
| $r$ | Pay-scaling parameter | Determines the rate at which mean income (within a firm) increases by hierarchical level. | Specific to each firm. |
| $\bar{I}_h$ | Mean pay in base hierarchical level | Sets the base level income of the firm, which determines firm average pay. | Specific to each firm. |
| $\sigma$ | Intra-hierarchical level pay dispersion parameter | Determines the level of inequality within hierarchical levels of a firm. | Identical for all firms. |

### E.1 Span of Control Parameters

The parameters $a$ and $b$ together determine the shape of firm employment hierarchy. These parameters are estimated from an exponential regression on case-study data (Fig. 5A). The model assumes that parameters $a$ and $b$ are *constant* for all Compustat firms.

Because the case-study sample size is small, there is considerable uncertainty in these values. I incorporate this uncertainty into the model using the *bootstrap* method (Efron and Tibshirani 1994), which involves repeatedly resampling the case-study data (with replacement) and then estimating the parameters $a$ and $b$ from this resample. Figure 10 shows the probability density distribution resulting from this bootstrap analysis. I run the Compustat model many times, each time with parameters $a$ and $b$ determined by a bootstrap resample of case-study data.

### E.2 Base-Level Employment

Given span of control parameters $a$ and $b$, each Compustat firm hierarchy is constructed from the bottom hierarchical level up. Therefore, we must know base-level employment in each firm. However, the Compustat database provides data for *total* firm employment only. To estimate base-level employment, I use the model to re-

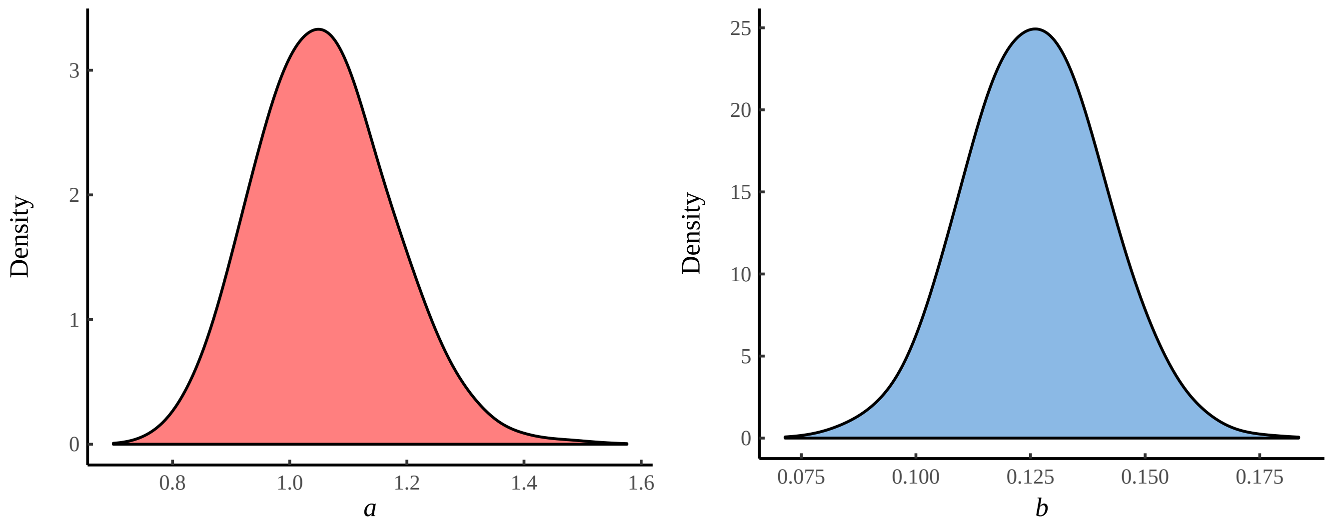


**Figure 10: Density Estimates for Span of Control Parameters**

This figure shows density estimates for the parameters $a$ and $b$, which together determine the 'shape' of the firm hierarchy. These parameters are determined from regressions on firm case-study data (Fig. 5). The density functions are estimated using a bootstrap analysis, which involves resampling (with replacement) the case-study data many times, and calculating the parameters $a$ and $b$ for each resample.

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verse engineer the problem. I input a range of different base employment values into equations 23, 26, and 28 and calculate total employment for each value. The result is a discrete mapping relating base-level employment to total employment. I then fit a high order polynomial to this relation. This function then allows us to predict base-level employment $E_1$, given total employment $E_T$, and span parameters $a$ and $b$.

### E.3 Pay-Scaling Parameter

The pay-scaling parameter $r$ determines the rate at which mean pay increases by hierarchical level. Unlike the span of control parameters, I allow the pay-scaling parameter to vary between firms. I fit the pay-scaling parameter $r$ to each Compustat firm using the CEO-to-average-employee pay ratio ($C$). The first step of this process is to build the employment hierarchy for each Compustat firm using parameters $a$, $b$, and $E_1$ (the latter is determined from total employment). Given this hierarchical employment structure, the CEO pay ratio in the modeled firm is uniquely determined by the parameter $r$. Thus, we simply choose $r$ such that the model produces a CEO pay ratio that is equivalent to the empirical ratio.

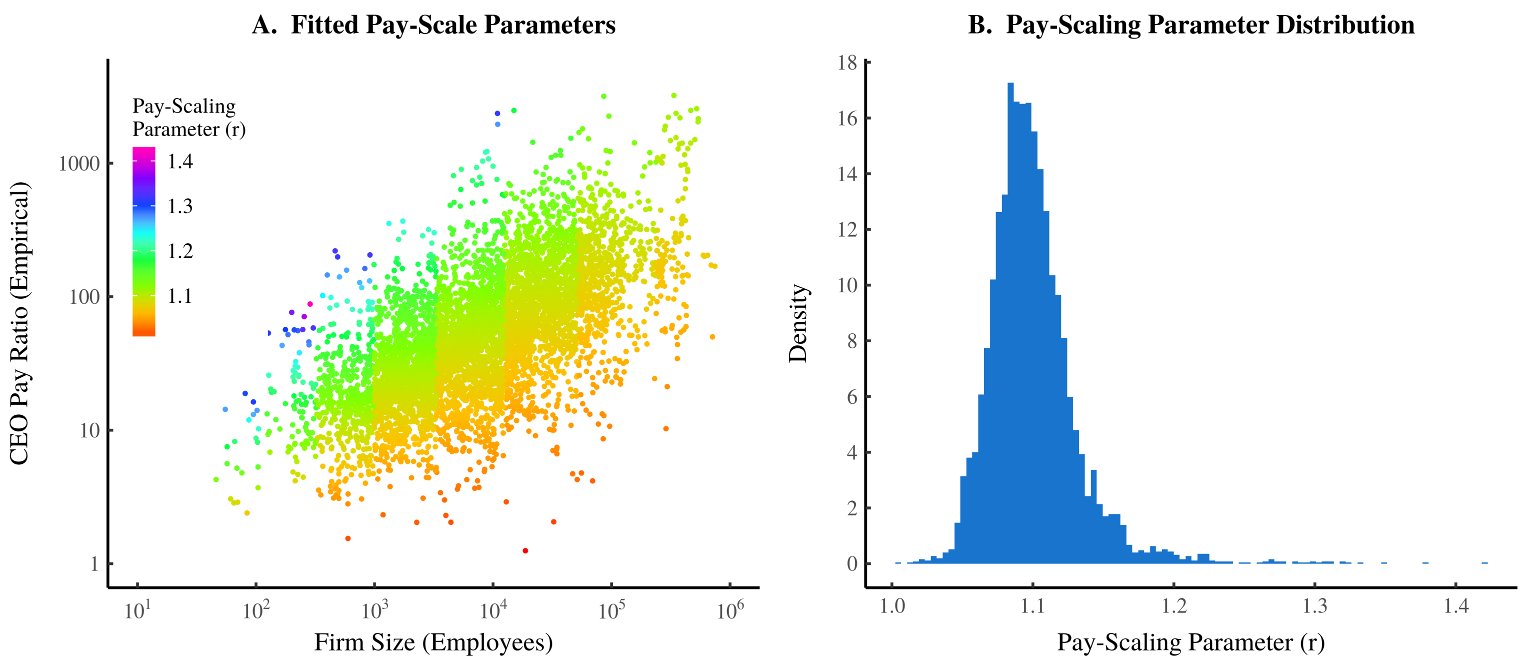


**Figure 11: Fitting Compustat Firms with a Pay-Scaling Parameter**

This figure shows the fitted pay-scaling parameters ($r$) for all Compustat firms. Panel A shows the relation between the CEO pay ratio and firm size, with the fitted pay-scaling parameter indicated by color. The discrete changes in color (evident as vertical lines) correspond to changes in the number of hierarchical levels within firms. The pay-scaling parameter distribution for all firms (and years) is shown in Panel B.

To solve for this $r$ value, I use numerical optimization to minimize the error function shown in Eq. 38. Here $C_{\text{Compustat}}$ and $C_{\text{model}}$ are Compustat and modeled CEO pay ratios, respectively.

$$\epsilon(r) = \left| C_{\text{model}} - C_{\text{Compustat}} \right| \tag{38}$$

For each firm, the fitted value of $r$ minimizes this error function. To ensure that there are no large errors, I discard Compustat firms for which the best-fit $r$ parameter produces an error that is larger than $\epsilon = 0.01$). Example fitted results for $r$ are shown in Figure 11.

## E.4 Base-Level Mean Pay

Having already fitted a hierarchical pay structure to each Compustat firm (in the process of finding $r$), we can use this data to estimate base pay for each firm. To do this, we set up a ratio between base-level pay ($\bar{I}_1$) and firm mean pay ($\bar{I}_T$) for both the model and Compustat data:

$$\frac{\bar{I}_1^{\text{Compustat}}}{\bar{I}_T^{\text{Compustat}}} = \frac{\bar{I}_1^{\text{model}}}{\bar{I}_T^{\text{model}}} \tag{39}$$

The modeled ratio between base pay and firm mean pay ($\bar{I}_1^{\text{model}}/\bar{I}_T^{\text{model}}$) is *independent* of the choice of base pay. This is because the modeled firm mean pay is actually a *function* of base pay (see Eq. 33 and 34). If we run the model with $\bar{I}_1^{\text{model}} = 1$, then Eq. 39 reduces to:

$$\frac{\bar{I}_1^{\text{Compustat}}}{\bar{I}_T^{\text{Compustat}}} = \frac{1}{\bar{I}_T^{\text{model}}} \tag{40}$$

We can then rearrange Eq. 40 to solve for an estimated base pay for each Compustat firm ($\bar{I}_1^{\text{Compustat}}$):

$$\bar{I}_1^{\text{Compustat}} = \frac{\bar{I}_T^{\text{Compustat}}}{\bar{I}_T^{\text{model}}} \tag{41}$$

### E.5 Intra-Hierarchical Level Income Dispersion

Intra-hierarchical level income dispersion within Compustat firms is modeled with a lognormal distribution, with the amount of inequality determined by the scale parameter $\sigma$. The model assumes that $\sigma$ is constant for all hierarchical levels within all firms.

I estimate $\sigma$ from the case-study data shown in Figure 5C. This data uses the Gini index as the metric for dispersion. To estimate $\sigma$, we first calculate the mean Gini index of all data ($\bar{G}$). We then use Eq. 42 to calculate the value $\sigma$. This equation corresponds to the lognormal scale parameter that would produce a lognormal distribution with an equivalent Gini index. This equation is derived from the definition of the Gini index of a lognormal distribution: $G = \text{erf}(\sigma/2)$.

$$\sigma = 2 \cdot \text{erf}^{-1}(\bar{G}) \tag{42}$$

Because the case-study sample size is small, there is considerable uncertainty in these values. I quantify this uncertainty using the *bootstrap* method (Efron and Tibshirani 1994), which involves repeatedly resampling the case-study data (with replacement) and then estimating the parameter $\sigma$ from this resampled data.

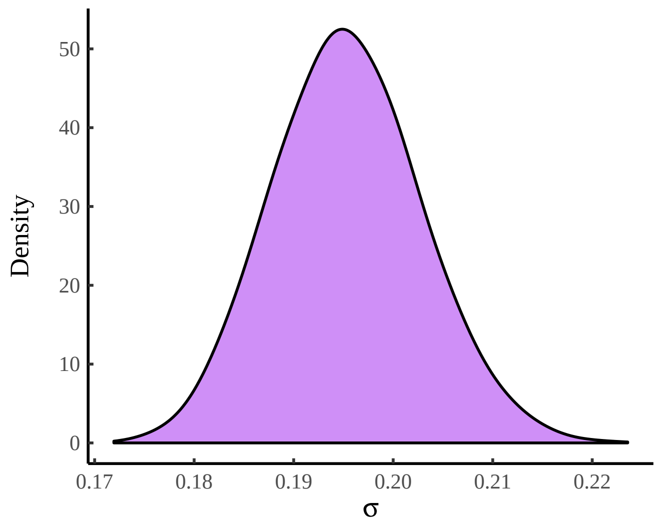


**Figure 12: Density estimates for Intra-Hierarchical Level Pay Dispersion Parameter $\sigma$**

This figure shows the distribution of the lognormal scale parameter $\sigma$, which determines pay dispersion within all hierarchical levels of all firms. The distribution is calculated using the bootstrap method.

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Figure 12 shows the probability density distribution resulting from this bootstrap analysis. In order to incorporate this uncertainty, I run the model many times, with each run using a different bootstrapped value for $\sigma$.

## E.6 Visualizing the Compustat Model

In order to aid the intuitive understanding of the Compustat model, Figure 13 visualizes the model in landscape form. Here I show selected firms from the year 2010. Each pyramid represents a separate firm with volume proportional to total employment. The vertical axis corresponds to hierarchical level. Income is indicated by color.

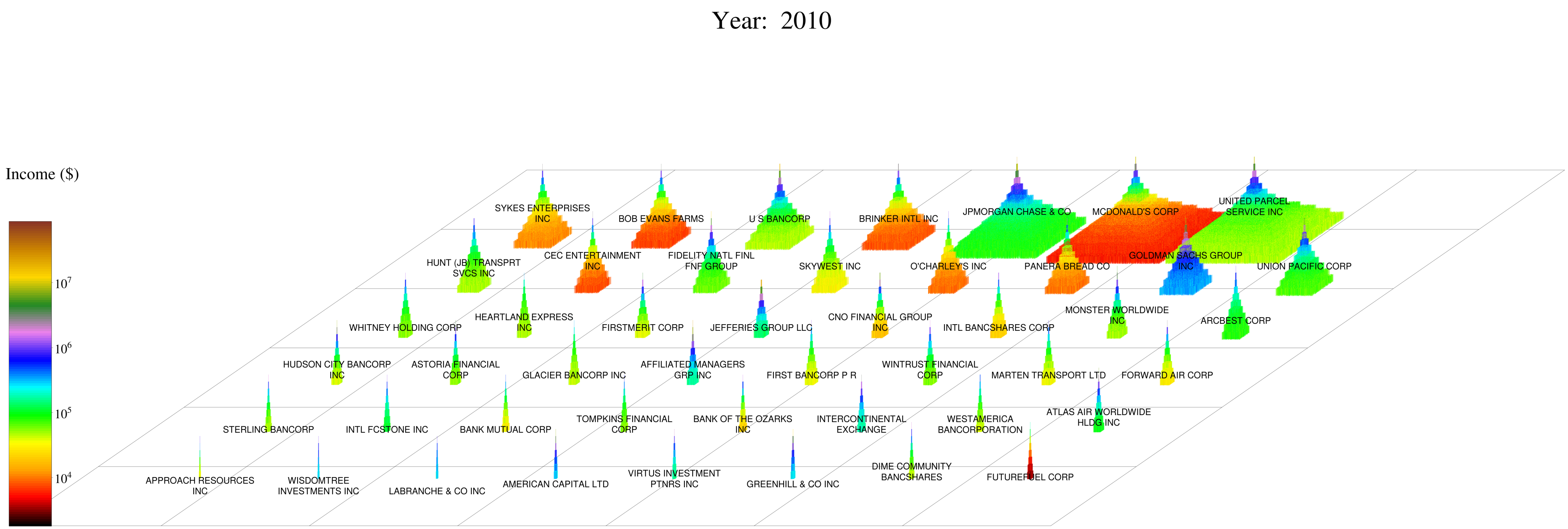


**Figure 13: Visualizing the Compustat Model**

This figure visualizes the results of the Compustat Model for selected US firms in the year 2010. Each pyramid represents a separate firm with volume proportional to total employment. The vertical axis corresponds to hierarchical level. Income is indicated by color.

# F Compustat Model Results

I review here the results of the Compustat model that are not discussed in the main paper. All results are generated using 5000 bootstrap model runs over different values for the parameters $a$, $b$, and $\sigma$. I review here the following: (1) estimates for income inequality within Compustat firms; (2) estimates for income by hierarchical level; and (3) aggregate inequality of all firms in the model.

## F.1 Inequality Within Compustat Firms

Figure 14 shows estimates of income inequality within Compustat firms. In Figure 14A, I illustrate how firm Gini indexes are related to both the CEO pay ratio and firm size. Note that the CEO pay ratio is a reliable indicator of firm inequality only for firms of the *same size*. A general feature of a hierarchical firm model is that when internal inequality is held constant, the CEO pay ratio nonetheless tends to increase with firm size (a feature first demonstrated by Herbert Simon (1957)). In Figure 14A, this feature is evident as color contours of constant within-firm Gini indexes that scale with both firm size and the CEO pay ratio.

Figure 14B shows the overall distribution of all firm Gini indexes. According to our model, 90% of Compustat firms have internal Gini indexes between 0.2 and 0.5. Note that the distribution is right-skewed — a small minority of firms have extremely unequal pay.

In Figure 14C I compare firm inequality in the Compustat model to inequality within the case-study firms discussed in Appendix B. The results indicate that Compustat firms are slightly more unequal than the case-study firms. However, because the case-study sample size is small, this difference is not statistically significant. A Kolmogorov-Smirnov test gives a p-value of 0.20, indicating that there is a reasonable (20%) probability that the two firm samples (model and case study) come from the *same* distribution. Thus, under the conventional 5% significance level, we cannot reject the null-hypothesis that these samples come from the same distribution.

Figure 14D shows the time evolution of average inequality within Compustat firms. During the late 1990s inequality rapidly increased, followed by relative stability from 2000 onward. While the *trend* is clear, there is significant uncertainty in the *absolute* level of inequality (as indicated by the shaded region). This uncertainty is due to the small case-study sample size on which key model parameters are based (see Appendix E).

Finally, Figure 15 shows Gini index estimates for the 50 most equal and 50

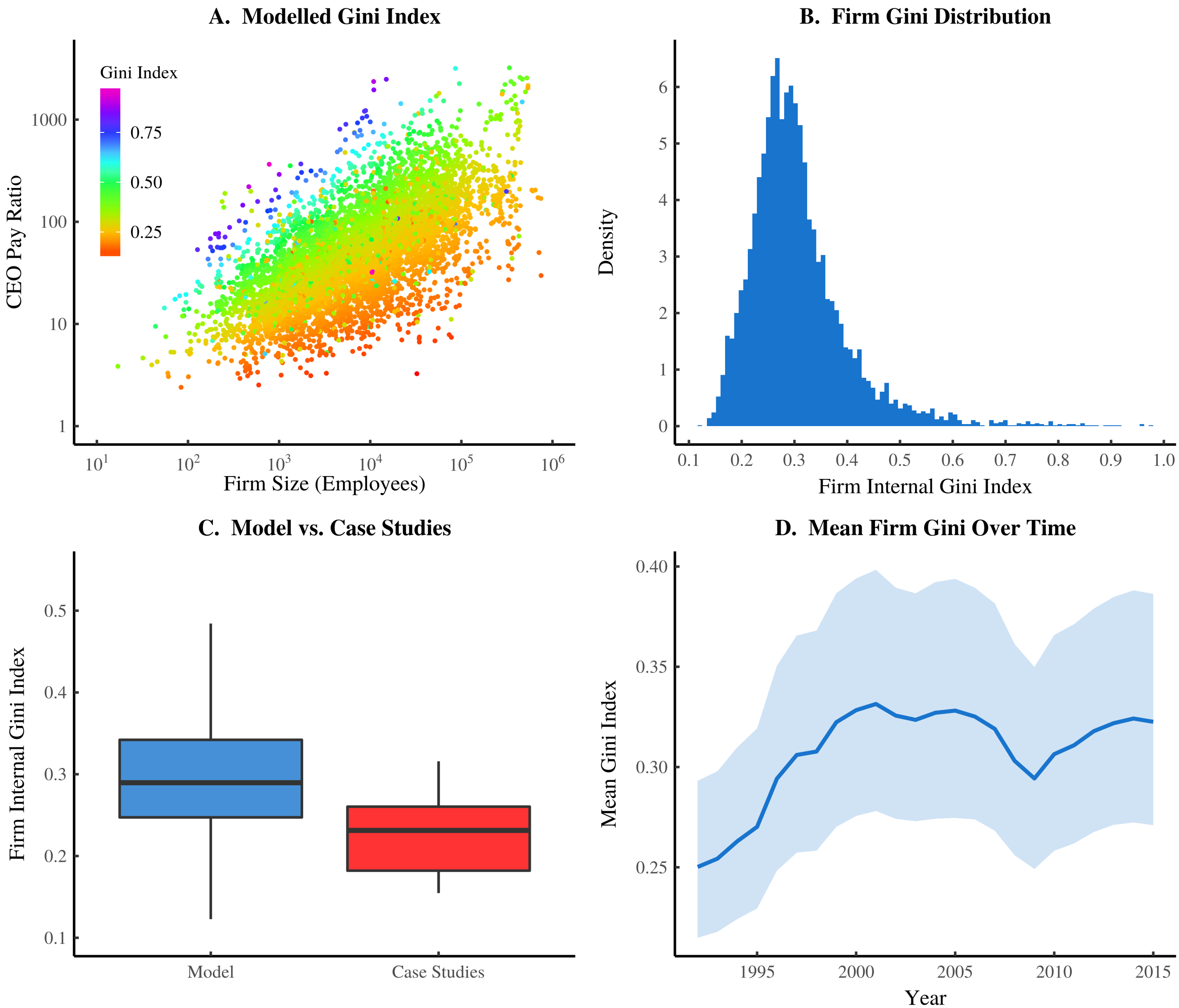


**Figure 14: Compustat Model Results for Intra-Firm Inequality**

This figure shows the firm internal Gini index results of the Compustat model. Panel A shows how firm internal inequality (indicated by color) is related to the CEO pay ratio and firm size. Panel B shows the distribution of modeled Gini indexes for all firms. Panel C compares model results to the Gini index of case-study firms (see section B.1 for case-study methods). Panel D shows time evolution of the average Gini index of all modeled firms. The shaded region indicates the 95% confidence interval. All results are computed from 5000 model runs, each with different bootstrapped parameters $a$, $b$, and $\sigma$.

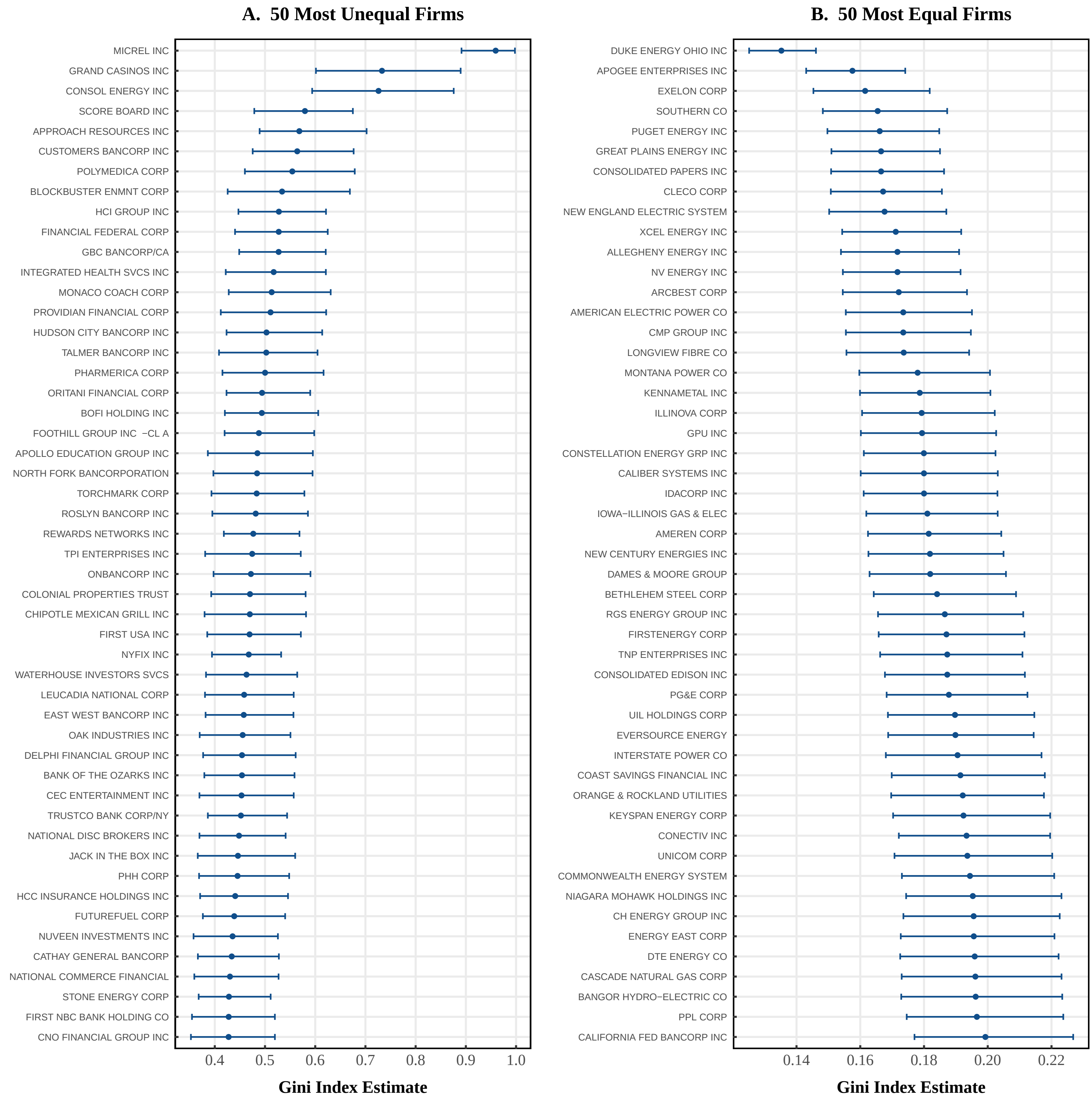


**Figure 15: The Most Equal and Unequal Compustat Firms**

This figure shows the 50 most unequal (Panel A) and 50 most equal firms (Panel B). Points indicate the mean Gini index for each firm, while the error bars show the 95% confidence interval calculated from 5000 bootstrap model runs.

most unequal firms. What is most interesting about these results is the *sectoral composition* of the 50 most equal firms. The vast majority (80%) are energy/utility companies. In the United States, firms in the utility sector are highly regulated, which leads to far more scrutiny over executive pay. Previous studies have found similar results — executives in regulated firms earn far less than those in unregulated firms (Joskow, Rose, Shepard et al. 1993). This finding has important implications for a hierarchical power theory of income distribution. It suggests that government regulation serves as a *check* on power, limiting the degree to which elites are able to use their status to amass wealth.

### F.2 Income By Hierarchical Level

The main purpose of the Compustat model is to estimate the income effect of grouping individuals by hierarchical level. Here I review summary statistics of the model's hierarchical pay structure. Figure 14A shows how mean income (across all firms in the model) increases by hierarchical level. The Compustat model's results are compared to the UK data documented by Mueller et al. (2016). In both the Compustat model and Mueller's data, mean income increases super-exponentially with hierarchical level — it increases *faster* than an exponential function. Figure 14B shows how intra-level income inequality changes by hierarchical level. For hierarchical levels 1-10, both the Compustat model and Mueller's data show similar trends.

The similarities between the model's results and Mueller's data lend credence to the model. However, what explains the differences? One key factor is that the Compustat data comes from the US, which has much more inequality than the United Kingdom, where Mueller's sample is taken. In this light, the differences in Figure 14A make sense. In the more unequal United States, income scales more rapidly with hierarchical level than in the United Kingdom. The results in Figure 14B can be similarly explained. In the more unequal United States, intra-hierarchical level income dispersion is greater than in the UK.

Another interesting result in Figure 14 is the conspicuous change in model trends for hierarchical levels above 11. Above this level, mean income no longer increases with hierarchical level. This may simply be an artifact of the particular Compustat firm sample. Going back to Figure 14A, note that the four largest firms have particularly low CEO pay ratios. Given the model's assumptions, only the very largest firms will have more than 11 hierarchical levels. Since the 4 largest firms have particularly low CEO pay ratios, resulting mean income in hierarchical levels 12-14 will be relatively low.

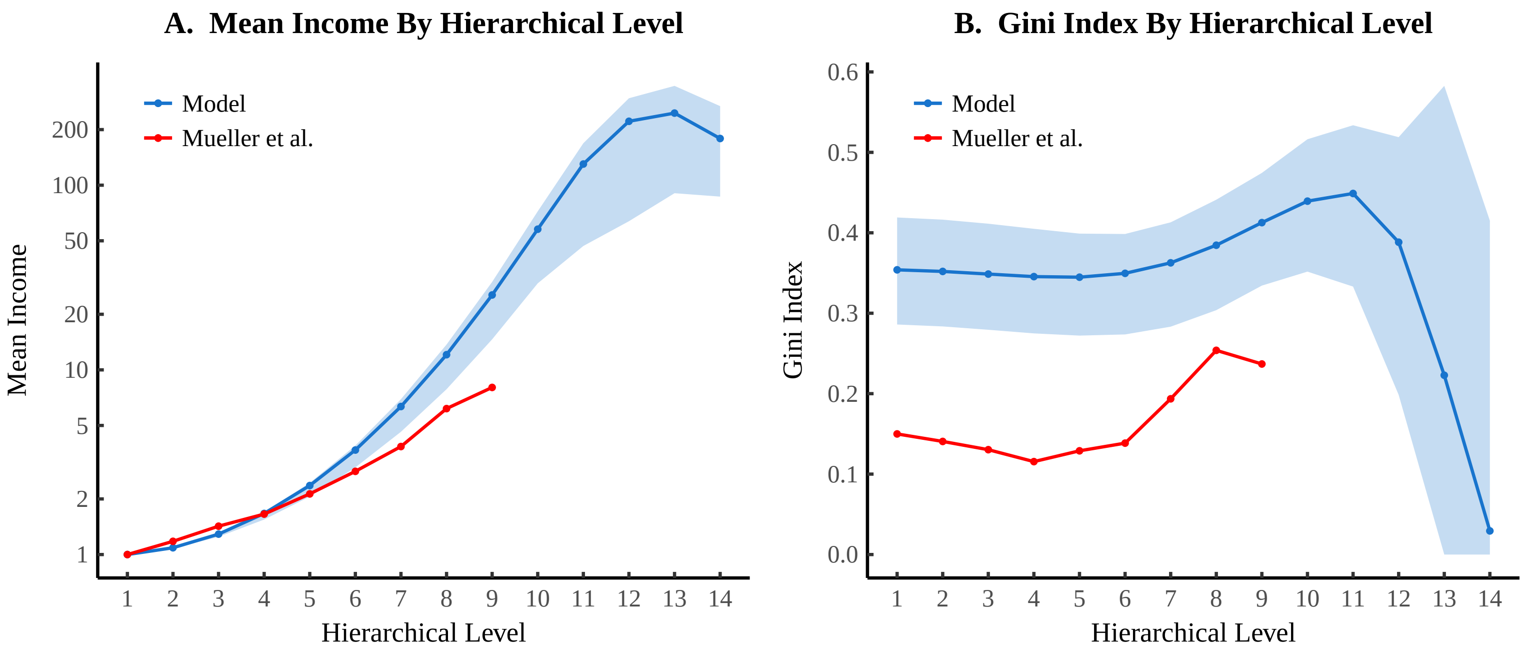


**Figure 16: Compustat Model Results for Income by Hierarchical Level**

This figure compares the results of the Compustat model to the UK data from Mueller et al. (2016). Panel A shows average income by hierarchical level (across all firms) indexed to pay in level 1. Panel B shows how intra-level inequality changes by hierarchical level. Shaded regions indicate the 95% confidence region of the model, estimated from 5000 bootstrap runs (see Appendix E).

What about the drop in intra-level inequality for hierarchical levels 12-14? This is a mathematical artifact. Individuals in upper hierarchical levels become exponentially rare. In some iterations of the Compustat model, there are only a few individuals in level 13, and only one in level 14. The low Gini index for these upper hierarchical levels results from the fact that a sample size of one has zero inequality, by definition.

## F.3 Aggregate Inequality

An important test of the Compustat model is to see if it produces aggregate levels of inequality that are comparable to US empirical data. Figure 17 shows the results of such a test. Here I plot the time-series trends in both US historical inequality and aggregate inequality in the Compustat model. This latter metric is calculated by aggregating (by year) all individuals in the model into a single sample, and then calculating the inequality of the resulting income distribution.

Figure 17A compares the model's aggregate Gini index against three different types of data published by the US Census: Gini by *individual*, *family*, and *household*. Two findings are evident. Firstly, the model is roughly consistent with the US empirical data over the period 2000-2015. However, the model produces too little inequality during the 1990s. Secondly, the US empirical data shows *contradictory* trends — roughly *constant* inequality among individuals, but secularly *increasing* inequality among families and households. The model reproduces the secular trend.

The problem with US Census data is that it is based on survey data, and individuals have a systemic tendency to under-report their incomes (especially if it is large). As an alternative to survey-based data, Thomas Piketty (2014) has focused on measuring inequality in the tail of the income distribution using income tax data. Figures 17B and 17C show Piketty's series for the top 10% and 1% income share in the United States. Both series show secularly increasing inequality over the period in question. The model reproduces these *trends* quite accurately, but at a lower absolute level of inequality.

Although the empirical data is itself contradictory, the important finding here is that the Compustat model produces a level of inequality that is consistent with official US data. This means that it is fair to compare the model's results to other results derived from official data.

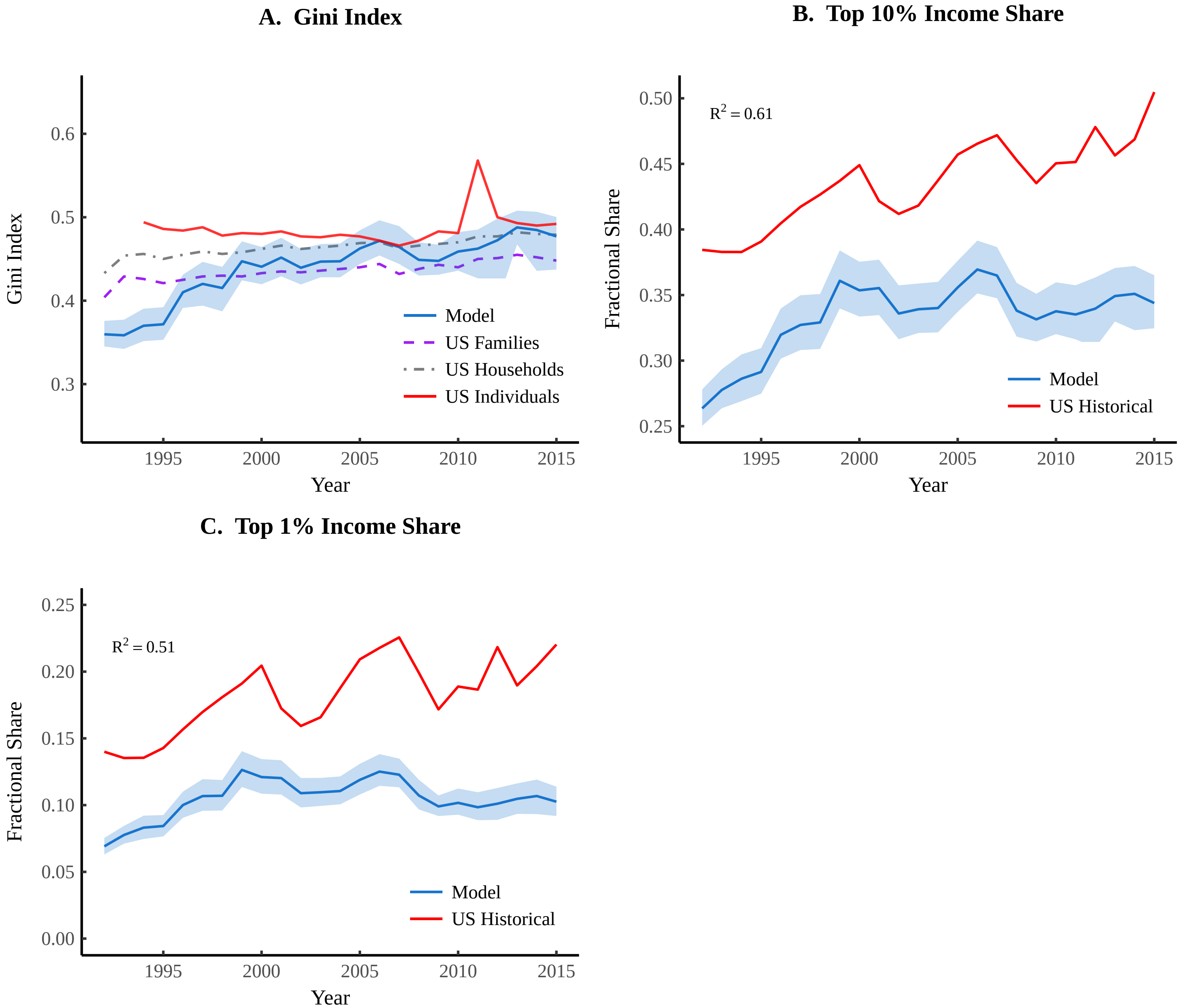


**Figure 17: Compustat Model Aggregate Inequality vs. US Historical Data**

This figure compares estimates of aggregate income inequality in the Compustat model to US historical data. Panel A compares the model Gini index to three different US measures (the Gini of individuals, families and households). Panel B shows the income share of the top 10%, while Panel C shows the top 1%. The shaded regions indicate the 95% confidence interval of the model, estimated over 5000 bootstrap runs. US Gini index data is for individuals, and comes from US Census table PINC-05. The 2011 outlier in US data is likely a statistical error. Families and Household Gini indexes are from the Federal Reserve Bank, series GINIALLRF and GINIALLRH, respectively. US top 10% and top 1% share data is from the World Wealth and Income Database, series *sptinc992j*.

# G A Sensitivity Analysis of the Compustat Model

The Compustat model relies on three parameters ($a$, $b$, and $\sigma$) that are determined from regressions on firm case-study data (see Appendix E). Parameters $a$ and $b$ define the span of control, and ultimately determine the 'shape' of each firm's hierarchy. The parameter $\sigma$ determines the level of income dispersion within each hierarchical level of a firm.

Because the case-study analysis contains only seven firms, there is a great deal of uncertainty in these parameters. Given this uncertainty, it is important to understand the 'sensitivity' of our model results to changes in the parameters $a$, $b$, and $\sigma$. I measure this sensitivity using a bootstrap analysis of the model's properties. I run the model many times, each time with a different resample of the case-study data (leading to different values of $a$, $b$, and $\sigma$). I use this bootstrapped data to analyze how each parameter affects the following metrics:

1. The signal-to-noise ratio of grouping individuals by hierarchical level;
2. The signal-to-noise ratio of grouping individuals by firms;
3. Aggregate levels of inequality within the entire model.

Figure 18 shows the results of this analysis. We can immediately conclude that the model is not sensitive to the value of $\sigma$, which has virtually no effect on any of the above metrics. However, the model appears to be highly sensitive to parameters $a$ and $b$. This sensitivity is least pronounced for the signal-to-noise ratio for grouping individuals by hierarchical level. However, for grouping by firms, changes in $a$ and $b$ have a strong effect on the signal-to-noise ratio.

This is an important finding. It suggests that our hierarchical level results (used to test the power-income hypothesis) are relatively robust. A different firm case-study sample would likely not lead to significant changes in our findings. Our firm results, however, are less robust. A different firm case-study sample could lead to very different results.

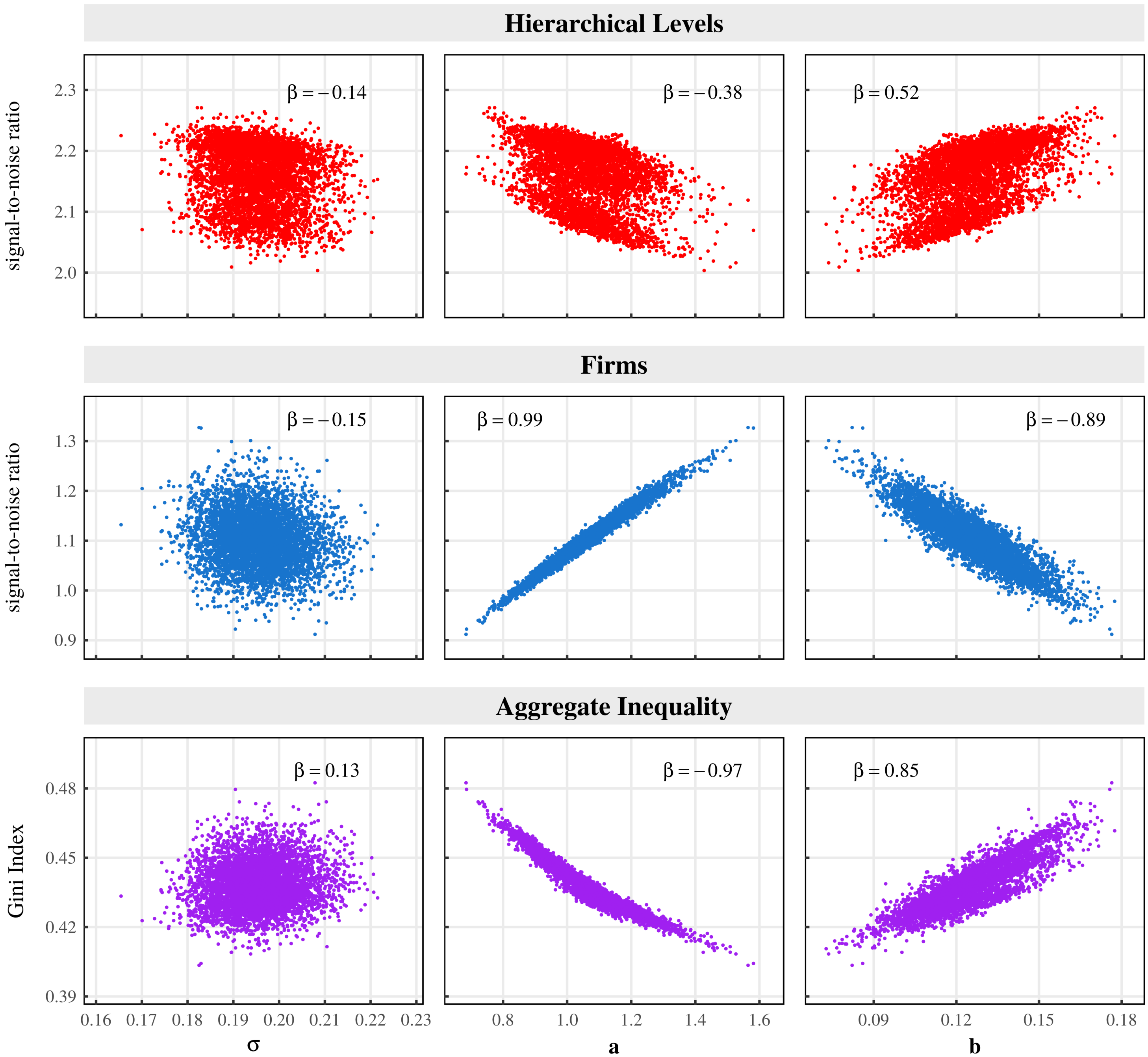


**Figure 18: A Sensitivity Analysis of Compustat Model Parameters**

This figure shows the results of a sensitivity analysis of Compustat model parameters. The top row shows the effect that each parameter has on the signal-to-noise ratio (using the Gini index) for grouping individuals by hierarchical level. The middle row shows the same for firms, and the bottom row for aggregate inequality in the model. Each plotted point indicates a different parameter combination. The normalized regression coefficient $\beta$ (which can range from -1 to 1) quantifies the model sensitivity.

## H Measuring Effect Size

In this section, I discuss what is meant by 'effect size' (in the context of this paper) and how the signal-to-noise ratio (using the Gini index) relates to more standard measures of effect size.

### What is Meant By 'Effect Size'

In the context of income distribution, there are two possible ways that we might define effect size:

**Definition A**: How much a factor affects *total inequality*.
**Definition B**: How much a factor affects *individual income*.

Effect size definition A refers to what we might call 'inequality accounting'. For instance, we might ask: how much do differences in pay between two groups contribute to total inequality? The point is that this definition attempts to measure how a given factor affects *total inequality*. In general, inequality accounting depends crucially on the *size* of the various groups.

Figure 19 shows this phenomenon. In both panels, groups A and B have equal differences in mean income and equal within-group income dispersion. However the total inequality obtained by merging the two groups varies dramatically depending on the relative size of A to B. In Fig. 19A, the two groups are of equal size, while in Fig. 19B, group B is 50 times smaller than group A. The resulting merger of A and B produces much more inequality when the two groups are of equal size than when they are not.

Effect size definition B is concerned only with the effect on *individual income*, not on accounting for total inequality. The key difference is that for definition A, we care about group size, while for definition B we do not. In more technical terms, effect size definition B should be calculated by drawing *equal sized samples* from each group. Perhaps the simplest and most intuitive metric of effect size definition B is *Cohen's d* (Eq. 43). This is defined as the difference in means ($\bar{x}$) between two group samples (A and B), divided by the within-group standard deviation ($s_W$).

$$d = \frac{\bar{x}_B - \bar{x}_A}{s_W} \tag{43}$$

Cohen's $d$ can be interpreted as a *signal-to-noise ratio*. The 'signal' is the difference in means (the effect we want to measure), while the 'noise' is the dispersion within groups, as measured by the standard deviation. In the case of income, the

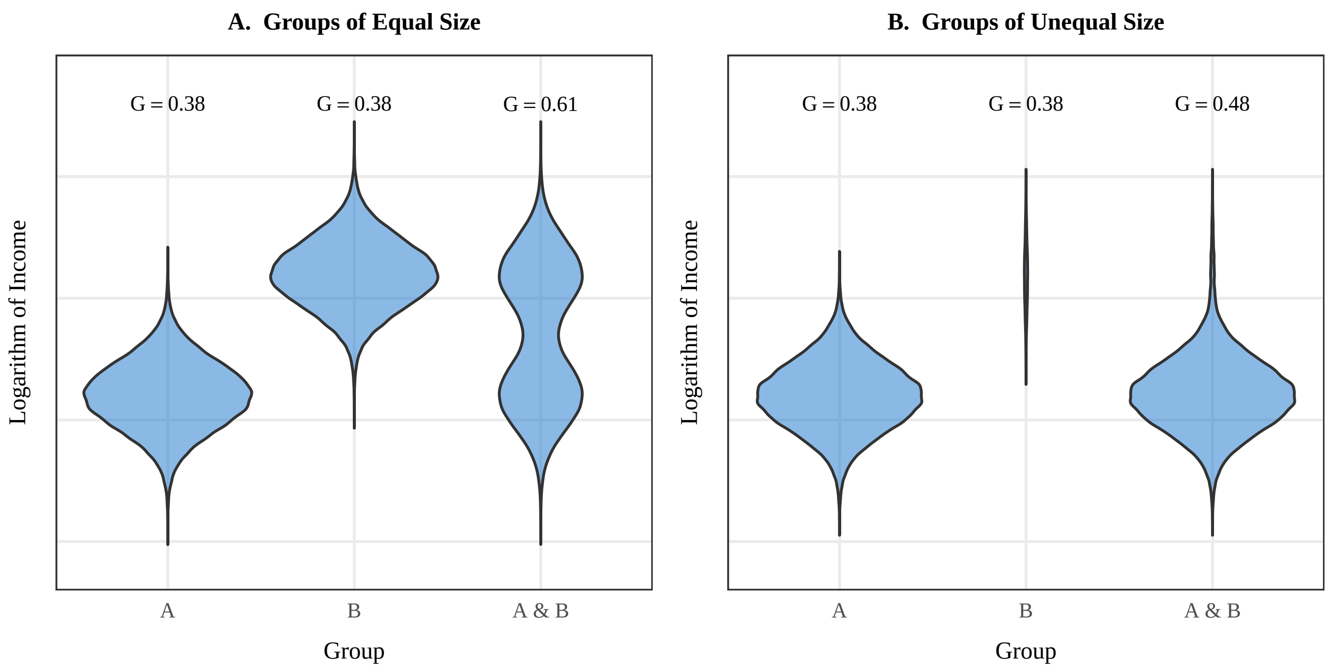


**Figure 19: How Group Size Affects Inequality Accounting**

This figure shows how differences in group size affect total inequality. In both panels, the income distribution of two different groups (A and B) are shown. The distributions are displayed as 'violin' plots, where the thickness of the violin indicates the number of individuals with that income. In both panels, groups A and B have identical differences in mean income, and identical within-group income dispersion (Gini indexes are shown above each violin). In the left panel, both groups have the same size. In the right panel, group B is 50 times smaller than group A. The rightmost violin plot in each panel shows the income distribution produced by merging groups A and B. Far more inequality is produced when the two groups are of equal size than when there are large differences in size.

size of the signal-to-noise ratio indicates how accurately we can predict someone's income based only on knowledge of their group membership (either A or B). The larger the signal-to-noise ratio, the more accurate the prediction.

In the example shown in Figure 19, group A and B have the same difference in means and the same within-group standard deviation in both the left and right panel. Therefore Cohen's $d$ would measure an identical effect size. To be clear, this is the effect on *individual income* (definition B), not the effect on inequality (definition A).

## The Signal-to-Noise Ratio

In this paper, I am concerned only with effect size definition B — the effect on individual income. I measure this effect size with a signal-to-noise ratio that uses the Gini index as a measure of dispersion:

$$G_{BW} = \frac{G_B}{\bar{G}_W} \tag{44}$$

Here $G_B$ is the Gini index of group means and $\bar{G}_W$ is the mean of all within-group Gini indexes.

How does this metric relate to more standard measures of effect size? It amounts to a signal-to-noise ratio that is similar to Cohen's $f^2$, the latter of which is a generalization of Cohen's $d$ to many different groups. To obtain Cohen's $f^2$, we divide variance between groups ($\sigma_B^2$) by average variance within groups ($\bar{\sigma_W^2}$):

$$f^2 = \frac{\sigma_B^2}{\bar{\sigma_W^2}} \tag{45}$$

Like Cohen's $d$, the $f^2$ metric is a signal-to-noise ratio. The 'signal' is the variance between groups, while the 'noise' is the variance within groups. (See Fleishman (1980) and Steiger (2004) for a more detailed discussion of the $f^2$ metric). Comparing the form of $f^2$ and $G_{BW}$, we see that the two measures of effect size are very similar. Both are signal-to-noise ratios, consisting of a ratio of between-group dispersion to within-group dispersion. Given this similarity, there should be some relation between the two measures.

Rather than attempt to show this similarity analytically, I use simulated data. I build a model based on the following assumptions:

1. Income within groups is lognormally distributed.

2. Within-group income dispersion is the same for all groups (but can vary over different model iterations).
3. Mean income between groups is lognormally distributed (and can vary between iterations).
4. Total inequality is (roughly) constant for all iterations.
5. The number of groups varies (between iterations) from 2 to 100.

For each iteration of the model, we define the mean income of each group by drawing randomly from a lognormal distribution. We then simulate individuals within each group by drawing randomly from (a different) lognormal distribution. The model has 2 key parameters: the lognormal scale parameter that defines the dispersion *between* groups, and the lognormal scale parameter that determines dispersion *within* groups. Varying these parameters changes the size of the group-income effect. For consistency, I use only parameter combinations that produce roughly the same level of total inequality (a Gini index of 0.5).

For each set of simulated data, I calculate both $G_{BW}$ and $f^2$. Because analysis of variance typically assumes that within-group data is normally distributed, I calculate $f^2$ using the *logarithm* of income. The results are shown in Figure 20. As expected, there is an extremely strong relation between the two effect-size measures. This indicates that an $f^2$ test of the power-income effect would likely give very similar results to the $G_{BW}$ findings shown in Figure 6 of the main paper. To reiterate, I do not conduct such an $f^2$ test in this paper because the relevant data is not available.

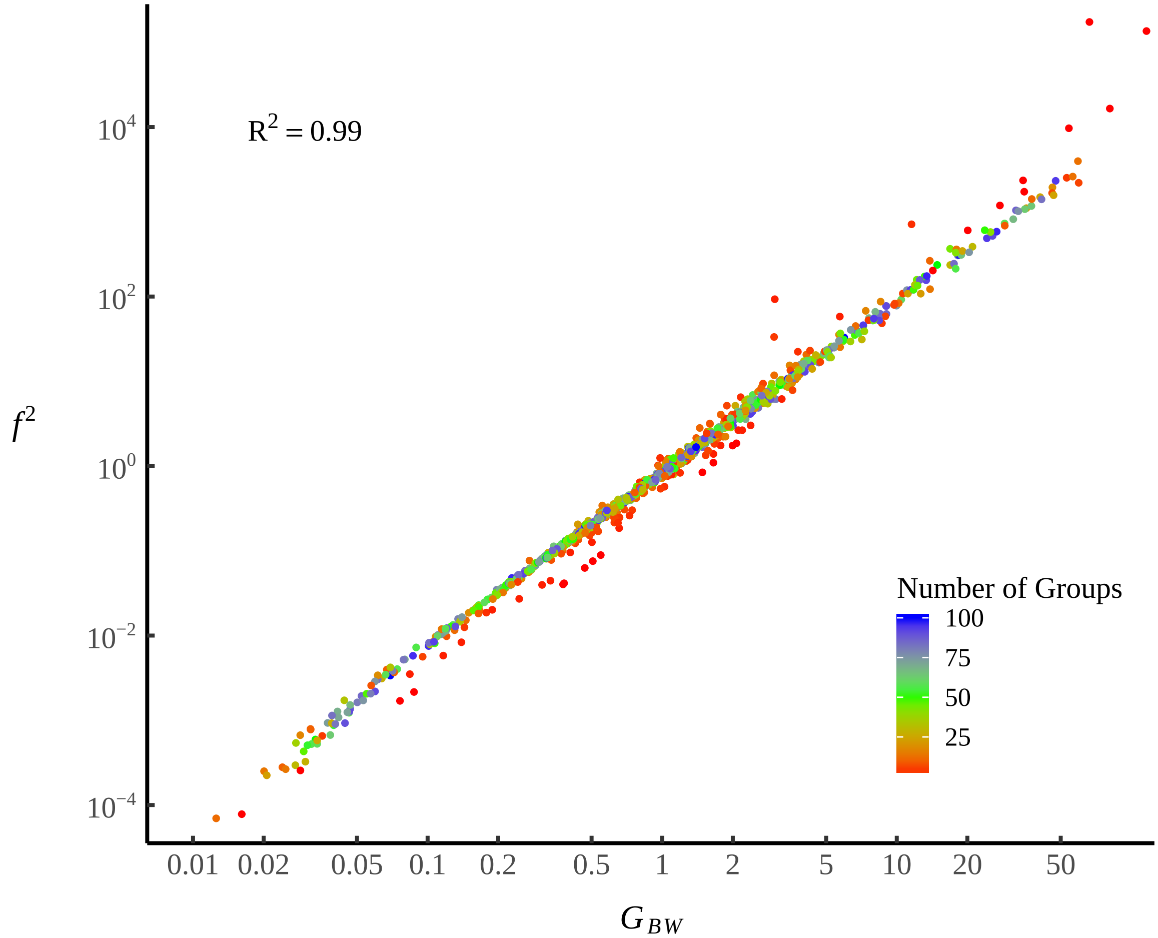


**Figure 20: Cohen's $f^2$ vs. the Gini Metric $G_{BW}$**

This figure compares Cohen's $f^2$ metric of effect size to my signal-to-noise Gini metric, $G_{BW}$. The comparison uses simulated data, and each data point represents different parameter combinations (see model assumptions above). Color indicates the number of groups used in each iteration.

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